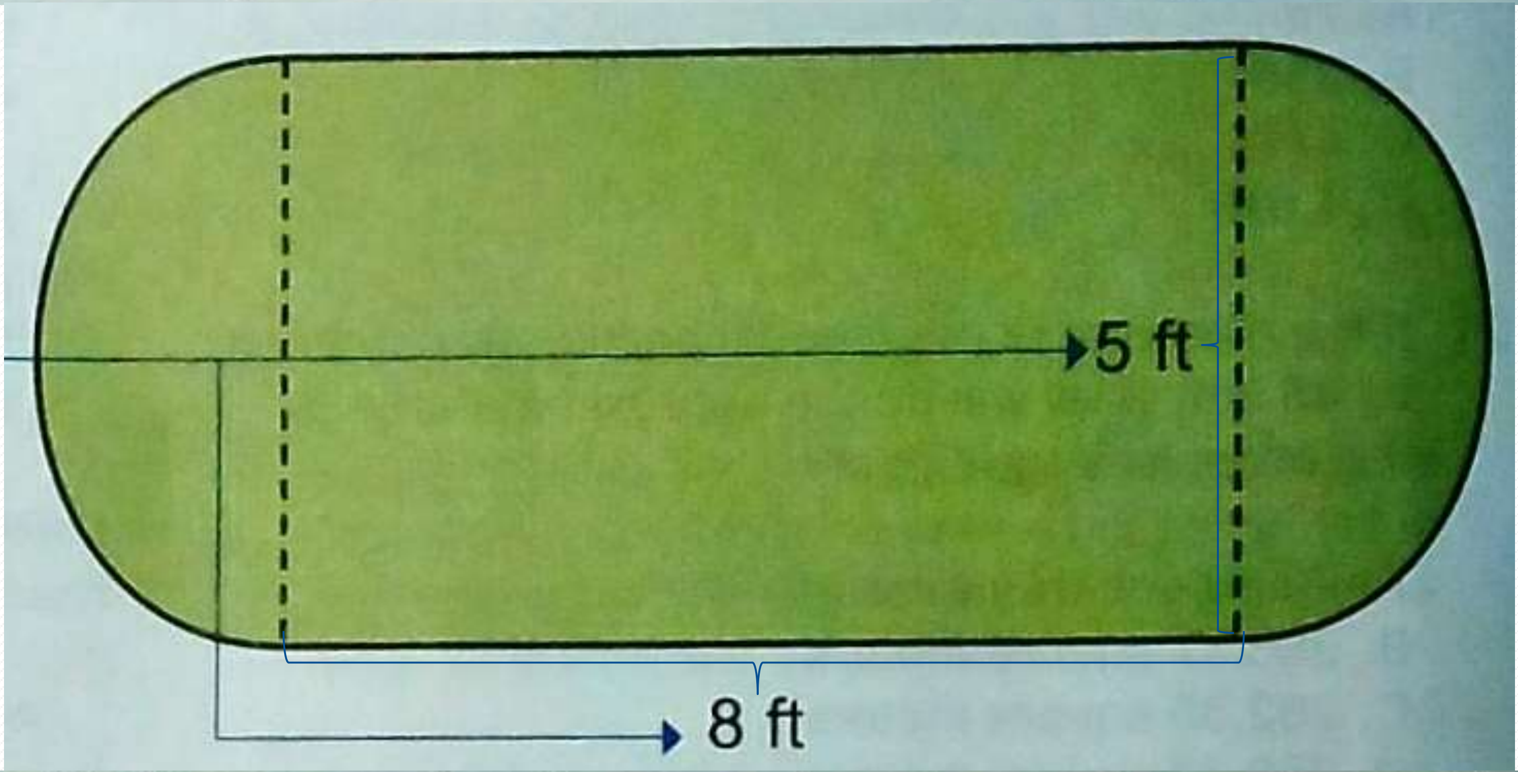
Composite Plane Figures (CPF)

A **composite plane figure** is made up of several shapes or parts of shapes. Composite plane figures can be 2- or 3-dimensional. You can find the perimeter or area of a composite plane figure shape by dividing it into simpler figures and adding the lengths of the sides or the areas of each simple figure.



2. What is the approximate area of each semi-circular region in the sample tablecloth?

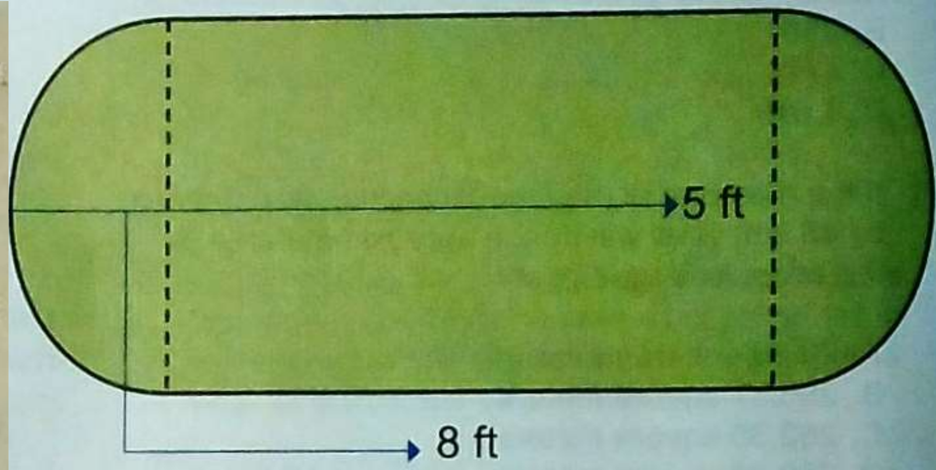
A. 19.63 ft²
B. 47.9 ft²
C. 55.7 ft²
D. 59.63 ft²

Use the formula for area (πr^2). The diameter of 5 ft ÷ 2 = a radius of 2.5 ft. So, (3.14)(2.5)(2.5) = an area of 19.625 ft², which rounds upward to 19.63 ft².

(I'll divide this by two, resulting in: 9.8 ft² as seen below)

Question 1 asks you to find the surface area of the actual tablecloth. Calculate numerical values for the actual product by first multiplying by 2. Next, calculate the area of each component shape to find the total area of this irregular figure. For example:

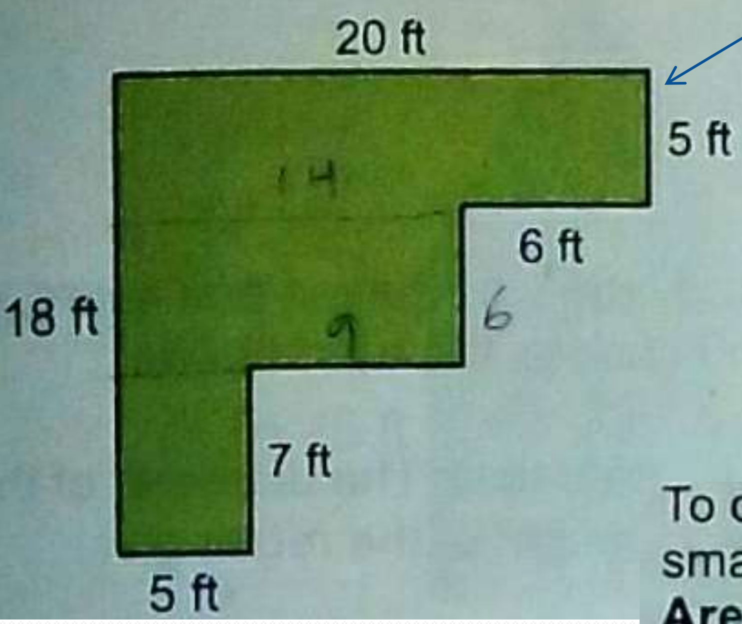
Total area = Area of semicircle + rectangle + semicircle



A. 40 ft^2
B. 78.5 ft^2
C. 179.63 ft^2
D. 238.5 ft^2

1. The image above represents a sketch of the top view of a sample tablecloth. If the actual tablecloth is twice as large as the sample, determine the approximate surface area of the actual tablecloth.

Note that since the actual tablecloth is twice as large as this sample, we need to multiply each value by 2 to get 10 ft and 16 ft. To find the area of the rectangular portion, use L × W so that A = 10 ft × 16 ft = 160 ft^2. Next, find the area of each semicircle by using the formula $\frac{1}{2}\pi r^2$. Multiply so that = $\frac{1}{2}(3.14)(5^2) = 39.25$. Since there are two semicircles, multiply this value by 2: 39.25 × 2 = 78.5 ft^2. Finally, add the areas of the figures = 160 ft^2 + 78.5 ft^2 = 238.5 ft^2.



Laura is carpeting her living room. How many square feet of carpet will she need?

A. 76
B. 189
C. 219
D. 317

To determine the area, this figure may be divided into 3 smaller rectangles running horizontally.

Area of rectangle #1 = L × W
= 20 ft × 5 ft = 100 ft²

Area of rectangle #2 = L × W
= 14 ft × 6 ft
= 84 ft²

Area of rectangle #3 = L × W
= 7 ft × 5 ft
= 35 ft²

Total area of figure = 100 ft² + 84 ft² + 35 ft² = 219 ft².

A. 72
B. 76
C. 78
D. 85

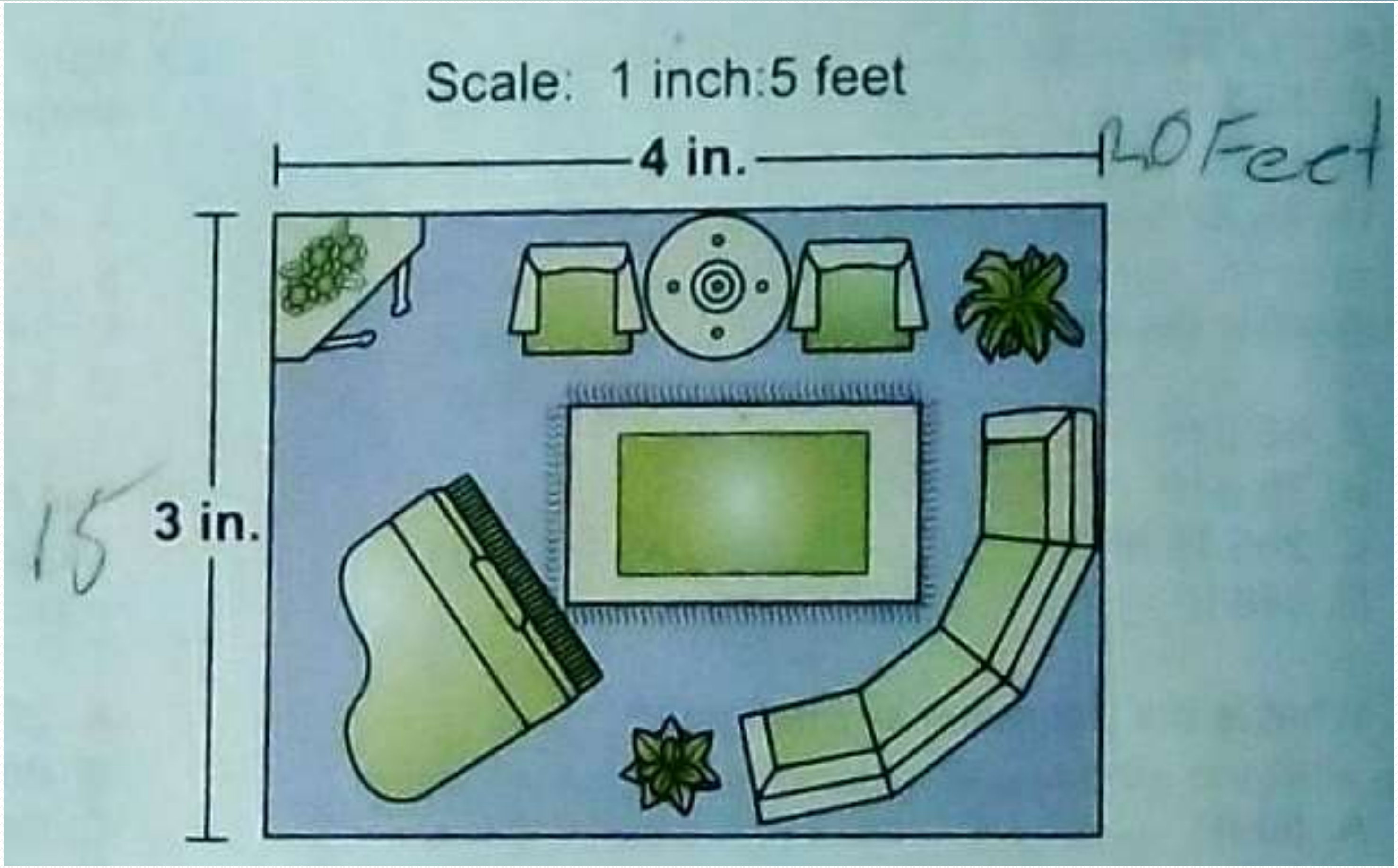
Laura needs tacking strips to go on the floor around the outer edge beneath the carpet. How many feet of tacking strips will she need? (This is just the perimeter)

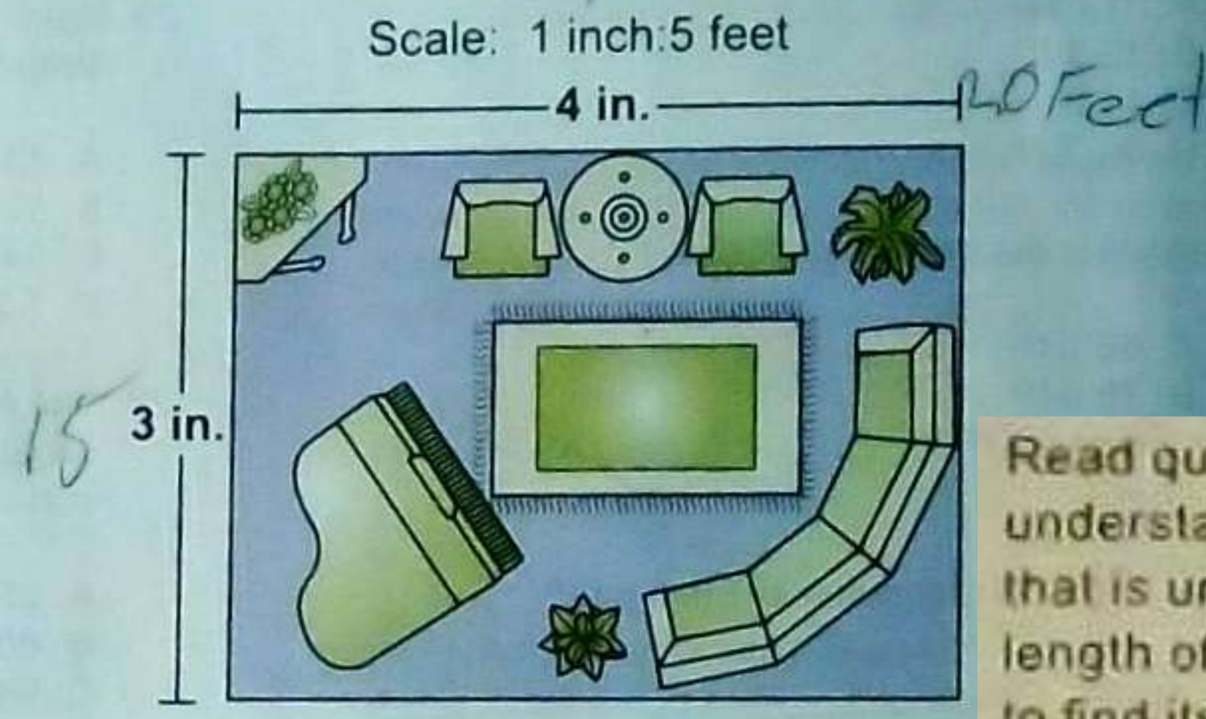
To determine the perimeter, add each side of the figure so that P = 5 ft + 6 ft + 6 ft + 9 ft + 7 ft + 5 ft + 18 ft + 20 ft = 76 ft.

Scale Drawings (SD)

Congruent figures are figures that are exactly the same shape and size because their corresponding angles and corresponding sides are equal. **Similar figures** are figures that have the same shape, but not the same size. Similar figures have equal corresponding angles, but the lengths of corresponding sides are proportional rather than equal.

Scale drawings, including maps and blueprints, are similar figures. A **scale factor** is the ratio of a dimension in a scale drawing to the corresponding dimension in an actual drawing or in reality. You can use ratios to determine the scale factor of a drawing. Proportions may be used to determine an unknown dimension in an actual or scale drawing, given the scale factor and the corresponding dimension.





Read question 1 carefully to understand the part of the proportion that is unknown. Here, you know the length of the actual object and need to find its length in the floor plan

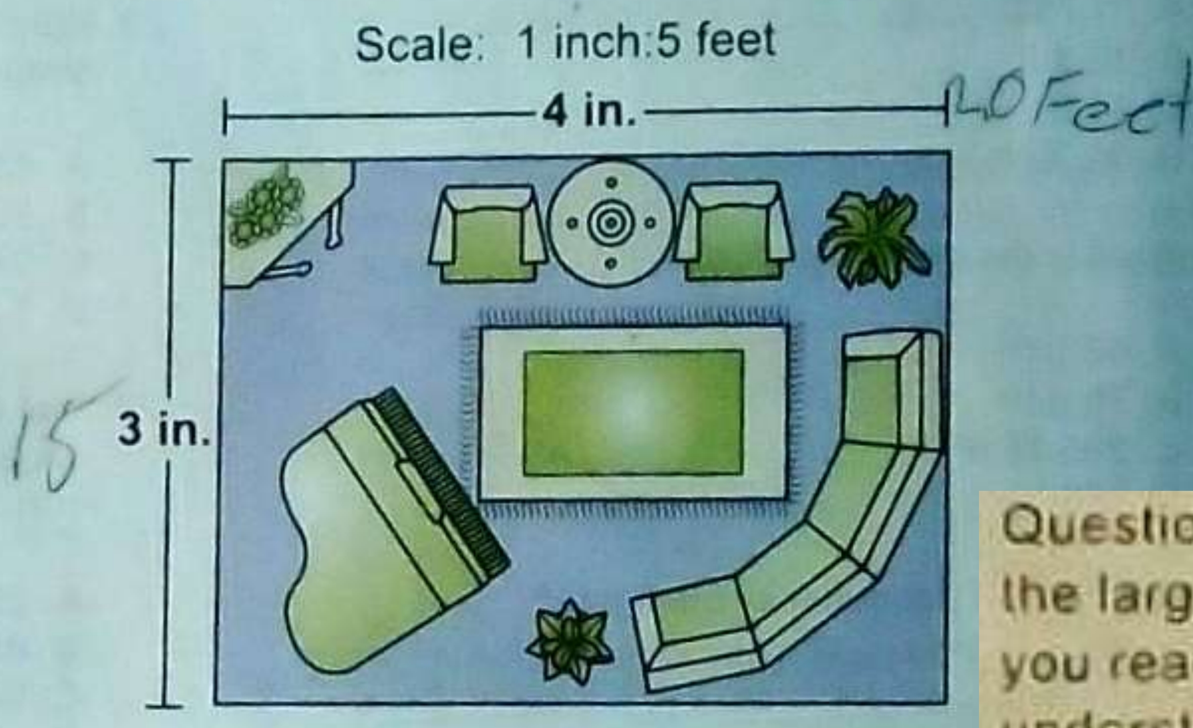
1. The length of the area rug in the actual living room is 8 feet. How many inches long is the area rug in the floor plan?

Set up a proportional relationship, and ensure you use the same units

$$\frac{\text{Actual rug}}{\text{Floor plan}} = \frac{5\text{ ft}}{1\text{ inches}} = \frac{8}{x}$$

$8 \div 5x = x = 1.6$ inches

A. 0.625 in.
B. 0.975 in.
C. 1.6 in.
D. 5.8 in.



Question 2 is seeking the length of the larger living room. Ensure that you read each question carefully to understand its meaning

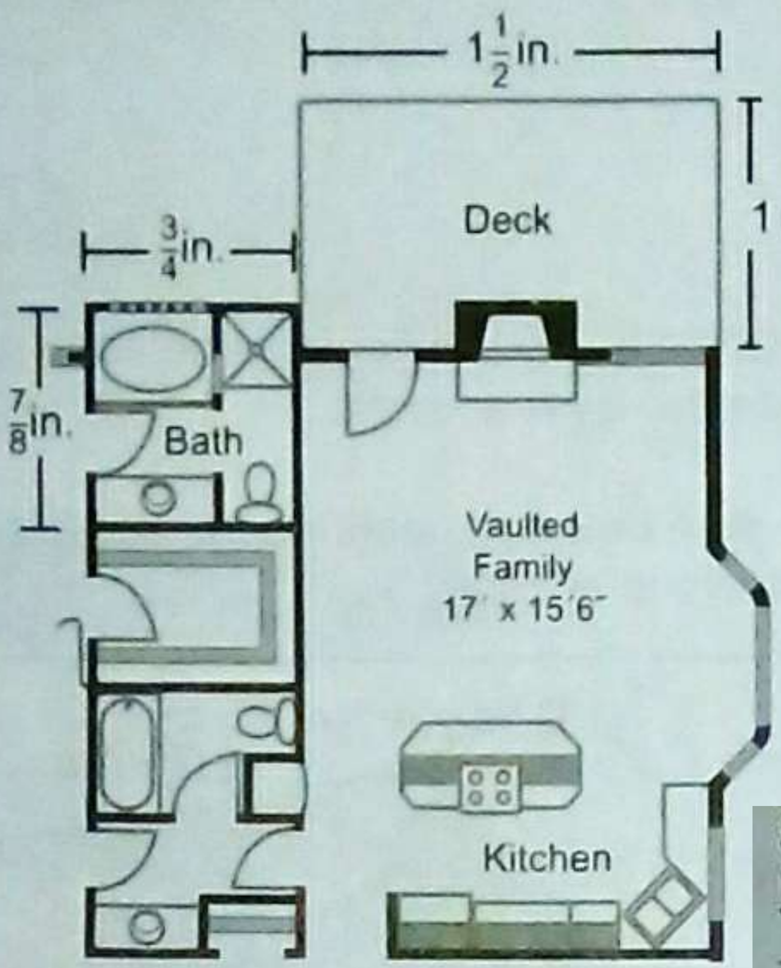
2. A similar floor plan will be used to build a larger living room in the basement. If the larger floor plan has a length of 3 inches, what is the length of the smaller living room?

$$\frac{\text{Actual length}}{\text{Floor plan}} = \frac{5 \text{ ft}}{1 \text{ inches}} = \frac{x}{3}$$

$x = 15$ ft

A. 3 feet
B. 15 feet
C. 20 feet
D. 24 feet

floor plan, the scale is 0.5 inch = 5 feet.



What are the dimensions of the actual deck?

A. 1.5 feet by 5 feet
B. 0.15 feet by 0.8 feet
C. 5 feet by 7.5 feet
D. 10 feet by 15 feet

Set up an equation to represent the relationship:

Dimension 1:

$$\frac{\text{Map}}{\text{Actual}} = \frac{0.5 \text{ inches}}{5 \text{ ft}} = \frac{1 \text{ inches}}{x}$$

So, 5 ft ÷ $0.5x = x = 10$ ft

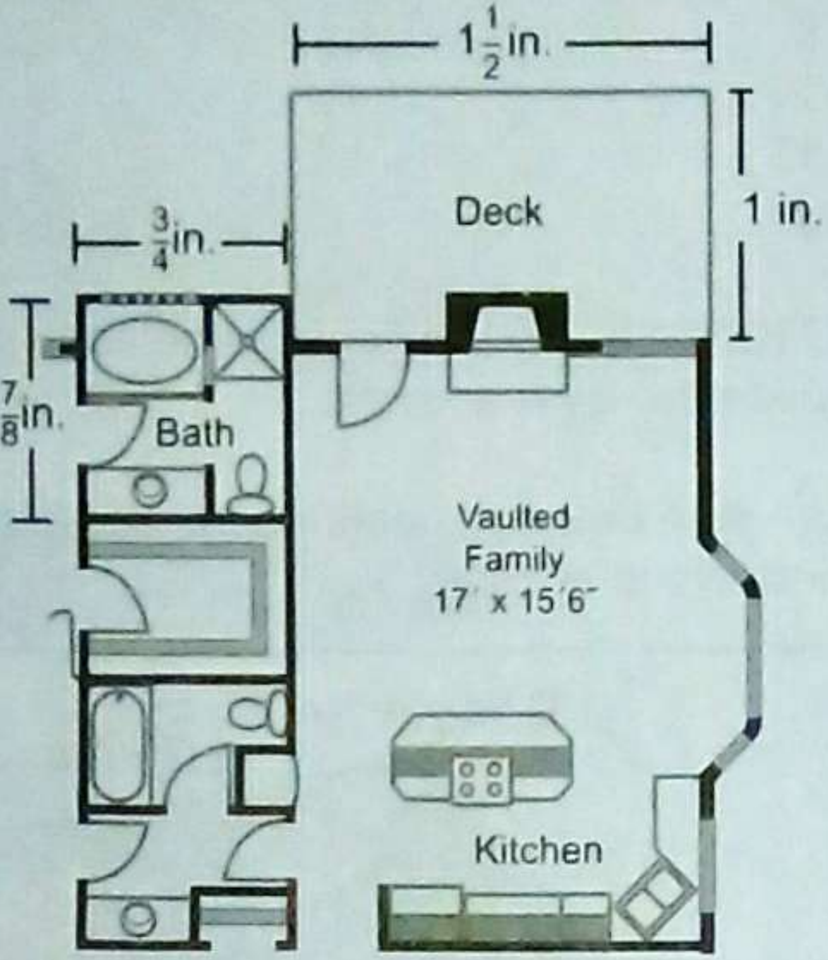
Dimension 2:

$$\frac{\text{Map}}{\text{Actual}} = \frac{0.5 \text{ inches}}{5 \text{ ft}} = \frac{1.5 \text{ inches}}{x}$$

So, 7.5 ft ÷ $0.5x = x = 15$ ft

The dimensions of the deck are 10 feet by 15 feet.

floor plan, the scale is 0.5 inch = 5 feet.



11. What is the actual length of the longer side of the bathroom?

A. $8\frac{3}{4}$ ft

B. $8\frac{1}{2}$ ft

C. 8 ft

D. $4\frac{3}{8}$ ft

Use the scale factor to set up a proposition and solve for x.

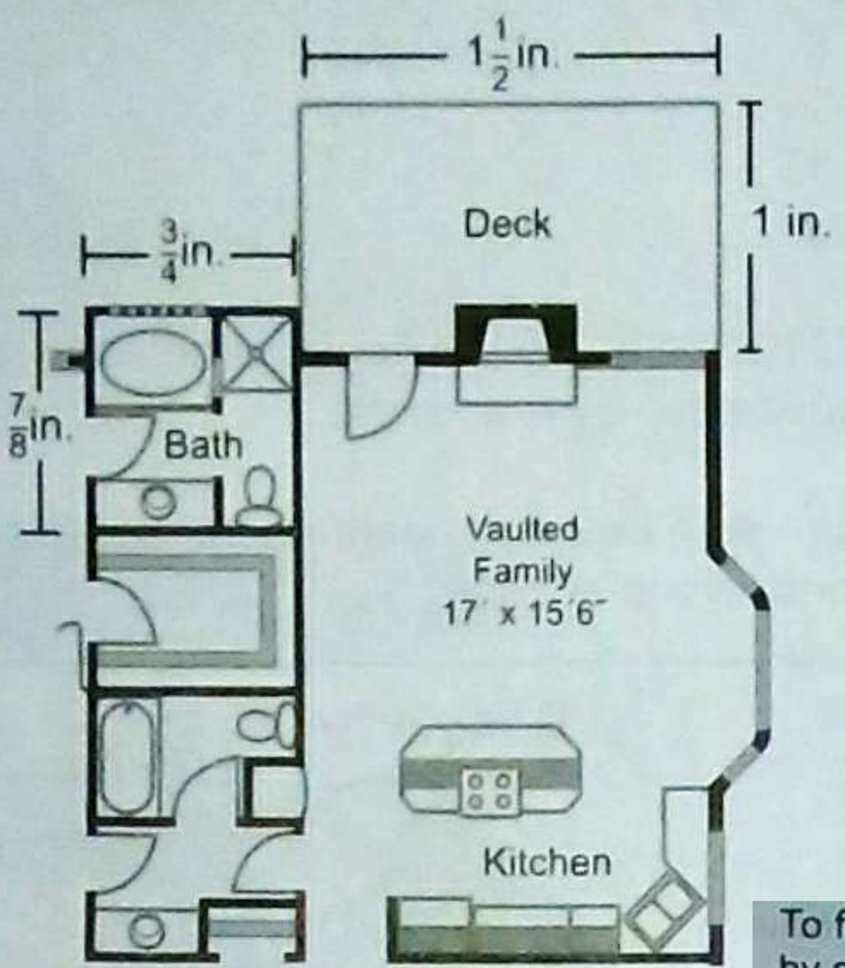
$\frac{\frac{7}{8}\ (0.875)}{x}$ or $= \frac{0.5}{x}$, so $0.5x = 4.375$

Map/Actual = 0.5 in/5 ft = (7/8) in/x ft = 8.75 ft

$x = 8.75$

The length of the longer side of the bathroom is $8\frac{3}{4}$ ft.

floor plan, the scale is 0.5 inch = 5 feet.



12. What are the dimensions of the actual bath?

A. $7\frac{1}{2}$ feet by $8\frac{3}{4}$ feet

B. $5\frac{1}{2}$ feet by $8\frac{1}{2}$ feet

C. 5 feet by 8 feet

D. 5 feet by $7\frac{1}{2}$ feet

Map/Actual = 0.5 in/5 ft = (3/4) in/x ft = 7.5 ft

Map/Actual = 0.5 in/5 ft = (7/8) in/x ft = 8.75 ft

$A = bh = (7.5 \text{ ft})\ (8.75 \text{ ft}) = 65.63 \text{ ft}^2$

To find the dimensions of the actual bathroom, find the width by setting up the following equation:

$$\frac{5 \text{ ft}}{0.5 \text{ inches}} = \frac{W \text{ ft}}{0.75 \text{ inches}}$$

So, 5 ft x .75 inches = 3.75 feet and $3.75 \div 0.5W = W = 7.5$ ft.

The width of the bathroom is $7\frac{1}{2}$ ft. Using the length calculated in question 11, the dimensions are $7\frac{1}{2}$ feet by $8\frac{3}{4}$ feet.

Prisms & Cylinders (P&C)

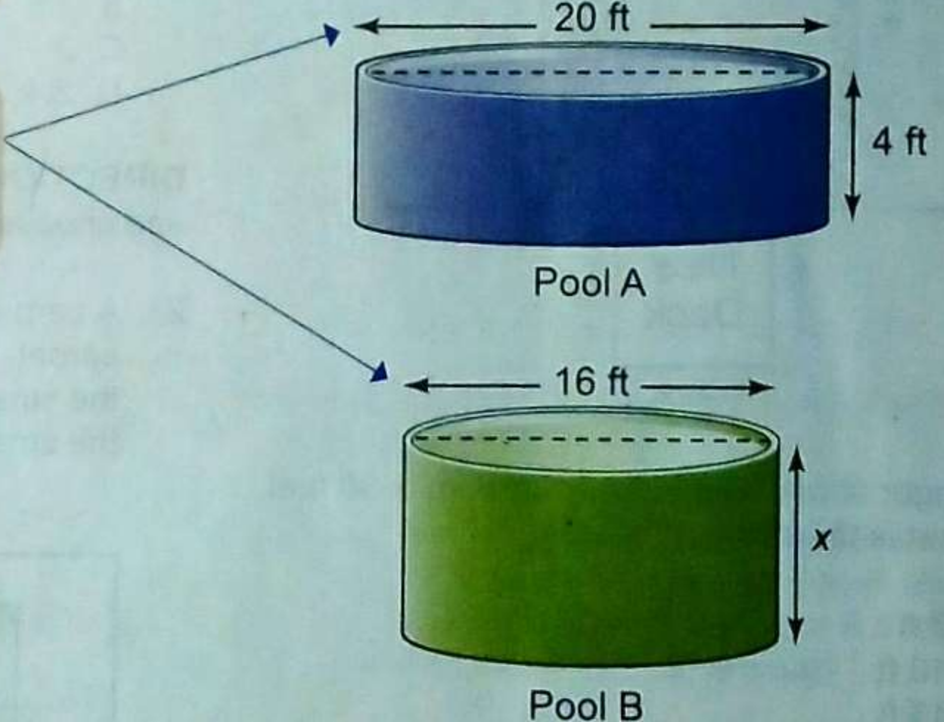
A **solid figure** is a 3-dimensional figure. The **volume** of a solid figure is the amount of space it occupies. The volume of a prism or cylinder is the product of the area of its base and its height. Volume is measured and shown in cubic units (^{3}).

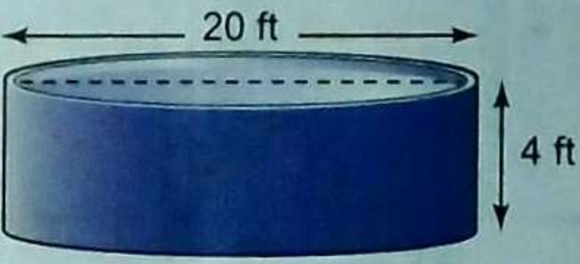
The **surface area** of a solid figure is the sum of the areas of its two bases *and* the area of its lateral surfaces. The surfaces of a prism are polygons. Use formulas for the areas of triangles, rectangles, and other polygons to compute the surface area of a prism. Such formulas will help you to determine the complete dimensions of a figure. For example, if you know the area of the base of a cylinder and it's surface area, you can calculate the height.

a The volume of the two pools is the same. For question 1, since the diameter and height are both given for Pool A, find the volume of this pool.

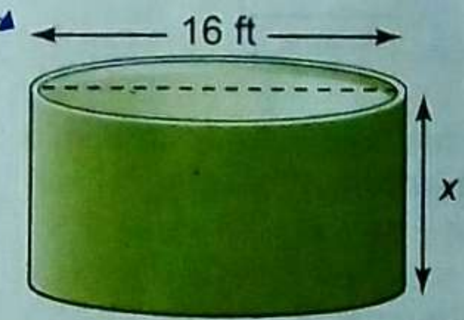
b Remember that the formula for volume of a cylinder uses the radius, not the diameter. For both questions, divide the diameter by 2 to find the radius.

Genevieve wants to buy an above-ground swimming pool. She is trying to decide between two models, shown below. Each model holds the same amount of water. The height of Pool B is represented by *x*.





Pool A



Pool B

1. What is the volume, to the nearest cubic foot, of each pool?

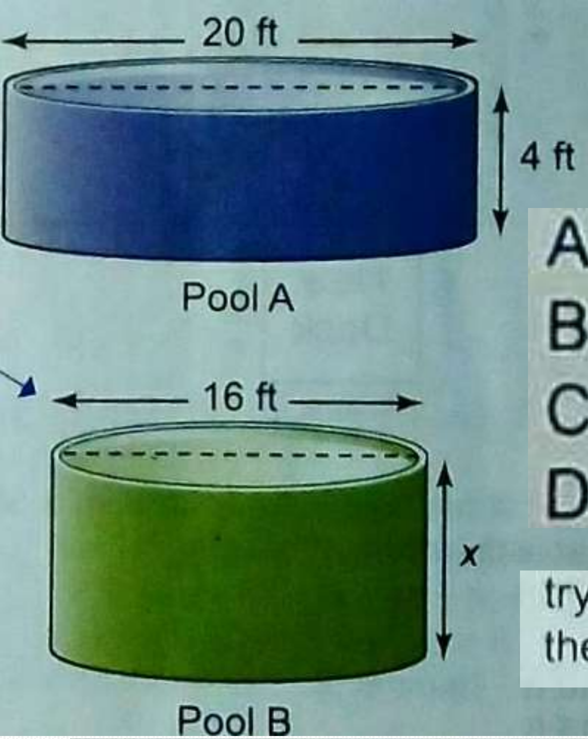
A. 400
B. 1,005
C. 1,256
D. 5,024

a The volume of the two pools is the same. For question 1, since the diameter and height are both given for Pool A, find the volume of this pool.

trying to decide between two models, shown below. Each model holds the same amount of water. The height of Pool B is represented by *x*.

b Remember that the formula for volume of a cylinder uses the radius, not the diameter. For both questions, divide the diameter by 2 to find the radius.

The volume of a cylinder is the product of the area of its base and its height, or $V = \pi r^2 h$. Since the diameter and height of Pool A are provided, find the volume of Pool A. The radius of Pool A is equal to $0.5 \times 20 = 10$ feet. Substitute 3.14 for π, 10 for r and 4 for h: $3.14 \times 10^2 \times 4 = 3.14 \times 100 \times 4 = 1{,}256$ ft.^{3} Answer choice A is the product of the square of the radius and the height. Answer choice B is result of using 4 as the radius, squaring it, multiplying by the diameter of the pool and then multiplying again by 3.14. Answer choice D is the result of using a radius of 20 feet.



2. What is the height, in feet, of Pool B?

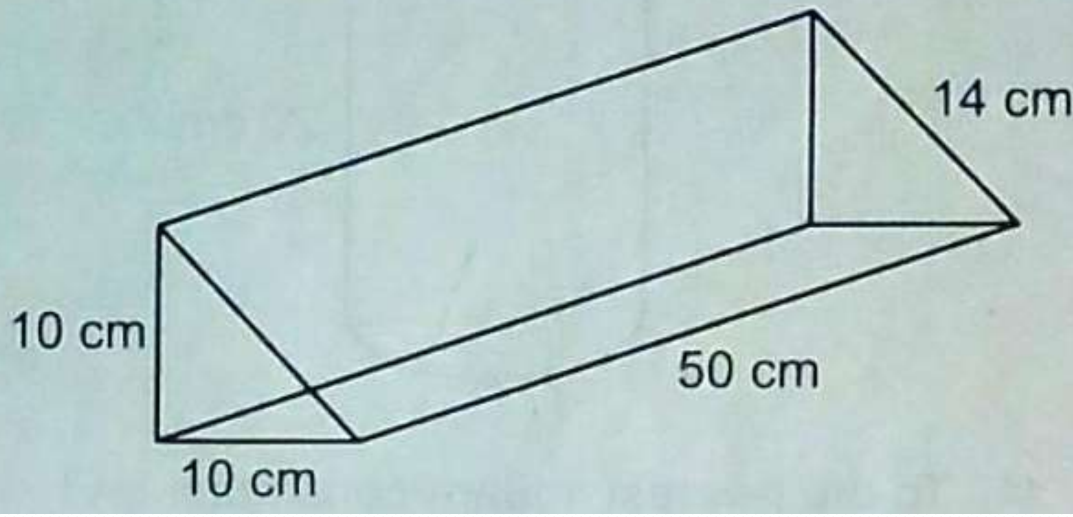
A. 4.25 ft
B. 6.25 ft
C. 8.0 ft
D. 10.0 ft

Remember that the formula for volume of a cylinder uses the radius, not the diameter. For both questions, divide the diameter by 2 to find the radius.

trying to decide between two models, shown below. Each model holds the same amount of water. The height of Pool B is represented by **x**.

The volume of a cylinder is the product of the area of its base and its height, or $V = \pi r^2 h$. To find the height of a cylinder with known volume and radius, divide the volume by πr^2. Since the volume of Pool B is equal to the volume of Pool A, $V = 1{,}256 \text{ ft}^3$. The radius of Pool B is half its diameter, or 8 feet. So, the height of the cylinder is $1{,}256 \div (3.14 \times 8^2) = 1{,}256 \div 200.96 = 6.25$ feet. The remaining answer choices are the result of errors in computing the volume or the height.

8. A shipping package has the shape of the triangular prism shown below.



A. 1,300 cm^2
B. 1,750 cm^2
C. 1,800 cm^2
D. 2,500 cm^2

Assuming the package has no gaps or overlaps, what area of cardboard is needed to produce the shipping package?

The amount of cardboard needed is equal to the surface area of the triangular prism. The prism has 2 triangular surfaces with base 10 cm and height 10 cm, 2 rectangular surfaces with length 50 cm and width 10 cm, and 1 rectangular surface with length 50 cm and width 14 cm.

So, the surface area is $2\left(\frac{1}{2}\right)(10)(10) + 2(10)(50) + 14(50)$.

Multiply: $100 + 1{,}000 + 700$ Add: 1,800 cm^2.

Pyramids, Cones & Spheres (PC&S)

A **pyramid** is a 3-dimensional figure with one polygon base and triangular faces. The volume, or amount of space a pyramid takes up, is $V = \frac{1}{3}Bh$. A **cone** has one circular base. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

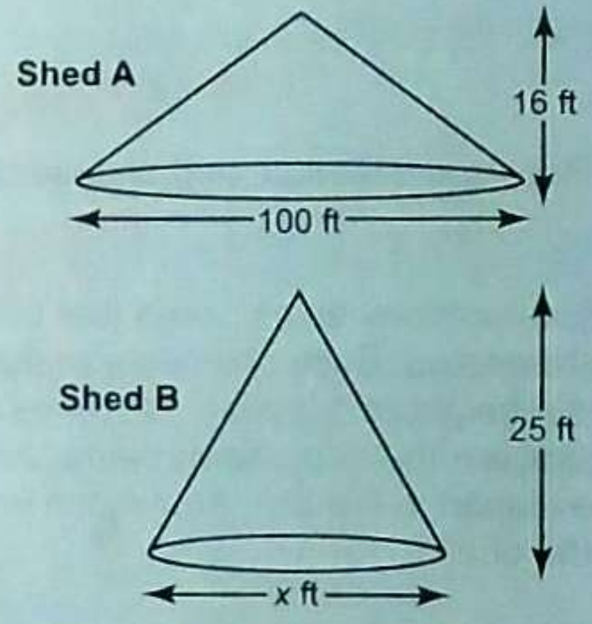
The **surface area** of a solid figure is the sum of the areas of surfaces. The surface area of a pyramid is the sum of the area of its base and its triangular faces. The area of each face is computed using its slant height. The formula for surface area of a pyramid is $SA = B + \frac{1}{2}Ps$, where B is the area of the base, P is the perimeter of the base, and s is the slant height. Meanwhile, the surface area of a cone is the sum of its circular base and its curved surface. The formula for surface area is $SA = \pi rs + \pi r^2$.

A **sphere** is shaped like a ball and has no bases or faces. The formula for volume of a sphere is $\frac{4}{3}\pi r^3$. The formula for surface area of a sphere is $4\pi r^2$.

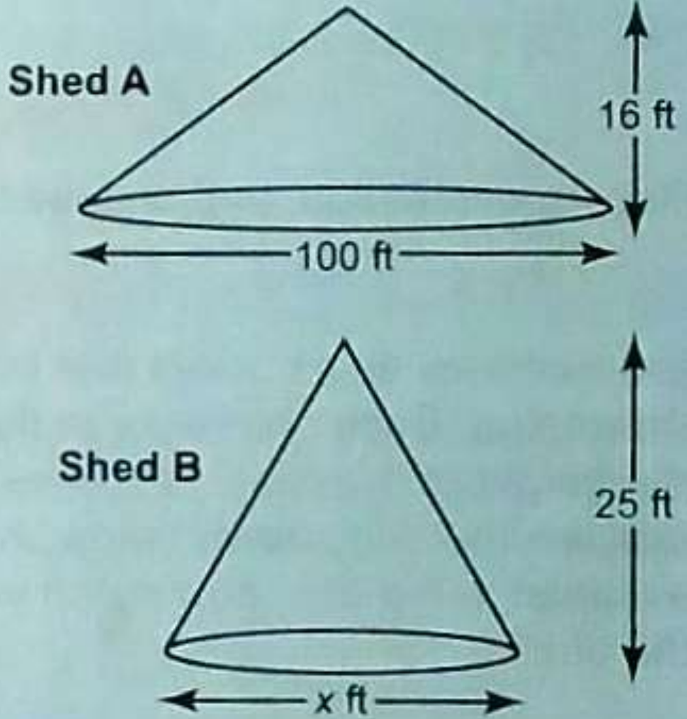
a The volume of the two buildings is the same. For question 1, since the diameter and height are both given for Shed A, you can find the volume of this shed.

b Remember that the formula for volume of a cone uses the radius, not the diameter. Divide the diameter by 2 to find the radius. Then multiply the calculated radius by 2 to find the diameter of Shed B.

A farmer is planning to build a conical grain storage shed on his property. To save on construction costs, he is choosing between two standard sizes, shown below. Each model holds the same amount of grain. The diameter of Shed B is represented by x.



A farmer is planning to build a conical grain storage shed on his property. To save on construction costs, he is choosing between two standard sizes, shown below. Each model holds the same amount of grain. The diameter of Shed B is represented by x.



What is the volume, to the nearest cubic foot, of each shed?

A. 41,867
B. 65,417
C. 125,600
D. 167,467

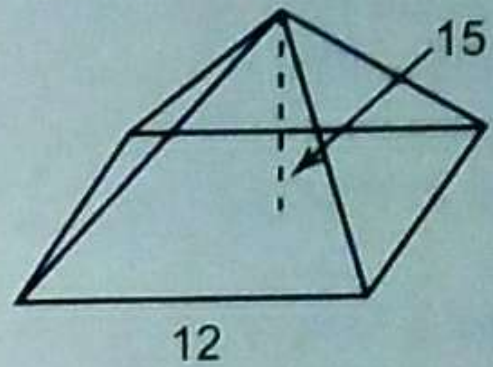
The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$. The diameter and height of Shed A are given, so use the diameter to find the radius and calculate the volume of Shed A: $r = 100 \div 2 = 50$ ft radius, so $V = \frac{1}{3} \times 3.14 \times 50^2 \times 16 = 41{,}866.66$ ft^3, which rounds to 41,867 ft^3.

b 2. What is the diameter, in feet, of Shed B?

To find the diameter of Shed B, use the volume to calculate the radius and then double the radius. The volume of Shed B is equal to the volume of Shed A: $V = \frac{1}{3} \times 3.14 \times 50^2 \times 16 =$ 41,867 ft^3. Substitute 41,867 for V and 25 for h in the formula $V = \frac{1}{3}\pi r^2 h$ and then solve for r: $41{,}867 = \frac{1}{3} \times 3.14 \times r^2 \times 25$. Multiply: $41{,}867 = 26.167r^2$. Divide: $r^2 = 1{,}600$. Take the square root of each side: $r = 40$ feet. So, the diameter is $40 \times 2 = 80$ feet.

A. 20
B. 40
C. 80
D. 160

13. What is the volume of the pyramid shown below?



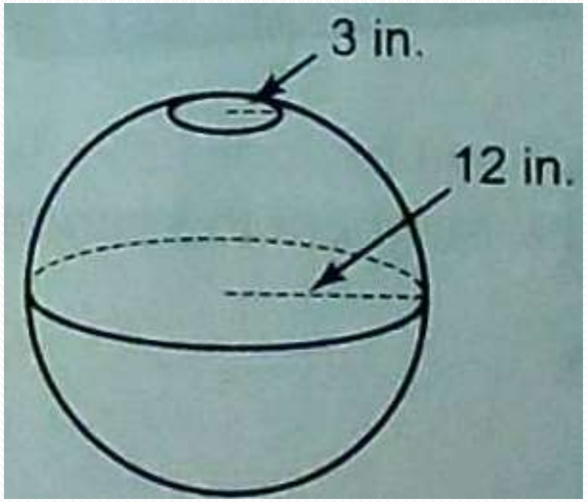
$$V = \frac{\square}{\square} \times \square \times \square = \square \text{ ft}^3$$

1	3	6	12	15	24	125	144	720

The volume of a prism is given by $V = \frac{1}{3}Bh$. The base edge is 12 feet, so the area of the base is $12^2 = 144$. The height is 15 feet. Substitute 144 for B and 15 for h and multiply to find the volume: $V = \frac{1}{3} \times 144 \times 15 = 720 \text{ ft}^3$.

A **sphere** is shaped like a ball and has no bases or faces. The formula for volume of a sphere is $\frac{4}{3}\pi r^3$. The formula for surface area of a sphere is $4\pi r^2$.

17. A spherical lantern has a circular hole cut out of its top. The radius of the lantern is 12 inches. The radius of the hole is 3 inches. About what area of paper is needed to construct the lantern?



A. 1,780 square inches
B. 1,810 square inches
C. 7,200 square inches
D. 7,230 square inches

The surface area of a sphere is $4\pi r^2$. The radius of the lantern is 12 inches, so the surface area is $4 \times 3.14 \times 12^2 = 1{,}808.64$ in.^{2} The area of a circle is πr^2, so the area of the circular hole is $3.14 \times 3^2 = 28.26$ in.^{2} Subtract the area of the hole from the surface area of the lantern: $1{,}808.64 - 28.26 = 1{,}780.38$ in.^{2}, which rounds to 1,780 in^2.

Composite Solids (CS)

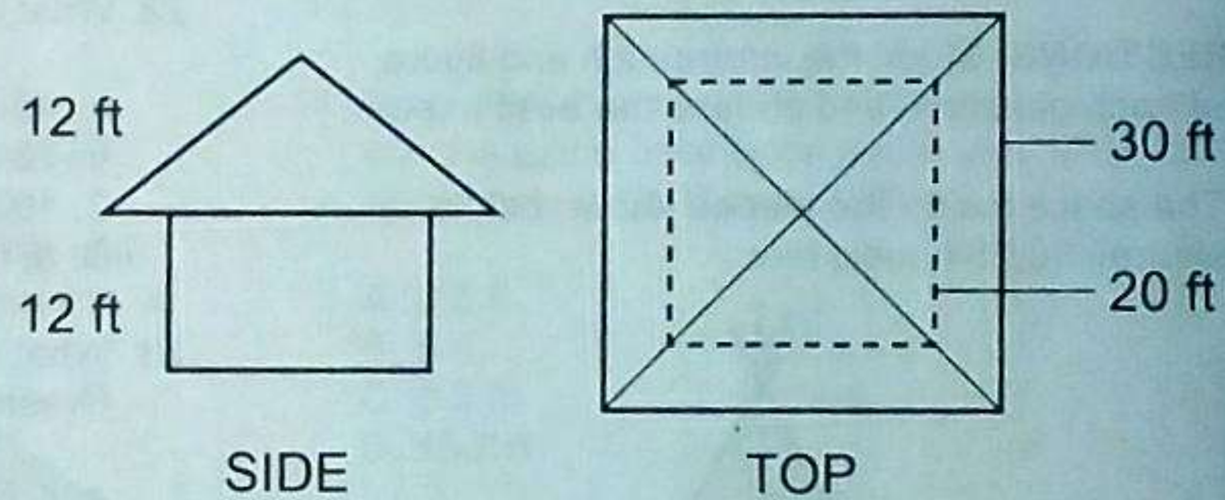
Calculations involving the surface areas and volumes of **composite solids** often can be made easier by breaking a composite solid into simpler shapes. In doing so, one must be careful to ensure the pieces recombine into the original shape.

Not all dimensions for simpler shapes may be given explicitly. Some may be inferred from the geometry and dimensions given for other elements of the whole. Relationships such as the Pythagorean Theorem also can be used to supply missing information.

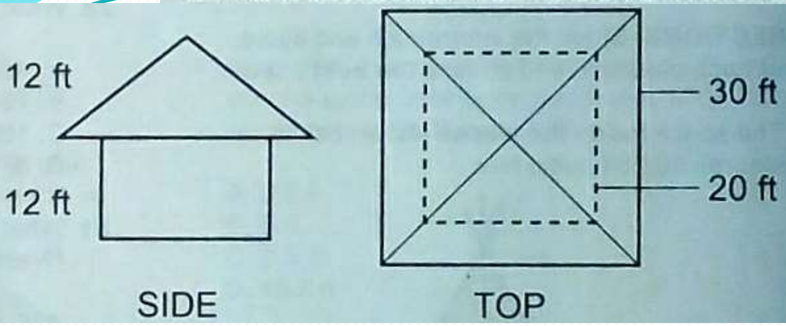
Volume calculations typically involve breaking a solid into simple shapes that have well-defined volume formulas. In this case, the solid is a combination of a rectangular prism and a square pyramid.

Problems involving the surface area of a composite solid must be read carefully to identify exactly the surfaces that are of interest. In this case, one might be interested in the walls of the building, but not the floor or the ceiling.

The following represents a storage shed, 20 ft by 20 ft, with 12-ft high walls. It is capped with a square pyramid, 30 ft by 30 ft, that adds an additional 12 feet to its height.



The following represents a storage shed, 20 ft by 20 ft, with 12-ft high walls. It is capped with a square pyramid, 30 ft by 30 ft, that adds an additional 12 feet to its height.



A. 6,400 cubic feet
B. 8,400 cubic feet
C. 10,200 cubic feet
D. 15,600 cubic feet

A. 400 cubic feet
B. 900 cubic feet
C. 4,800 cubic feet
D. 10,800 cubic feet

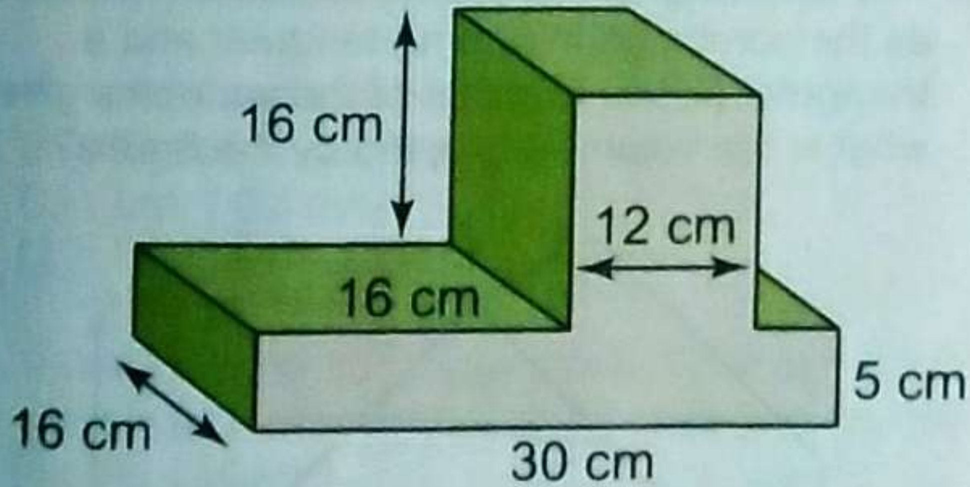
1. What is the volume occupied by the lower, square part of the shed?

2. What is the total volume occupied by the shed?

The lower part of the shed is a rectangular prism, 20 feet by 20 feet, and 12 feet high. The volume is the product of those three dimensions: (20)(20)(12) = 4,800 cubic feet (choice C).

The volume of the pyramid is $\left(\frac{1}{3}\right)Bh$, where B = 900 feet (the base is the product of s^2, or 30 × 30) and h = 12 feet. Substituting gives a volume of 3,600 cubic feet. Adding that to the volume of the lower part (4,800 cubic feet) gives a total of 8,400 cubic feet (choice B).

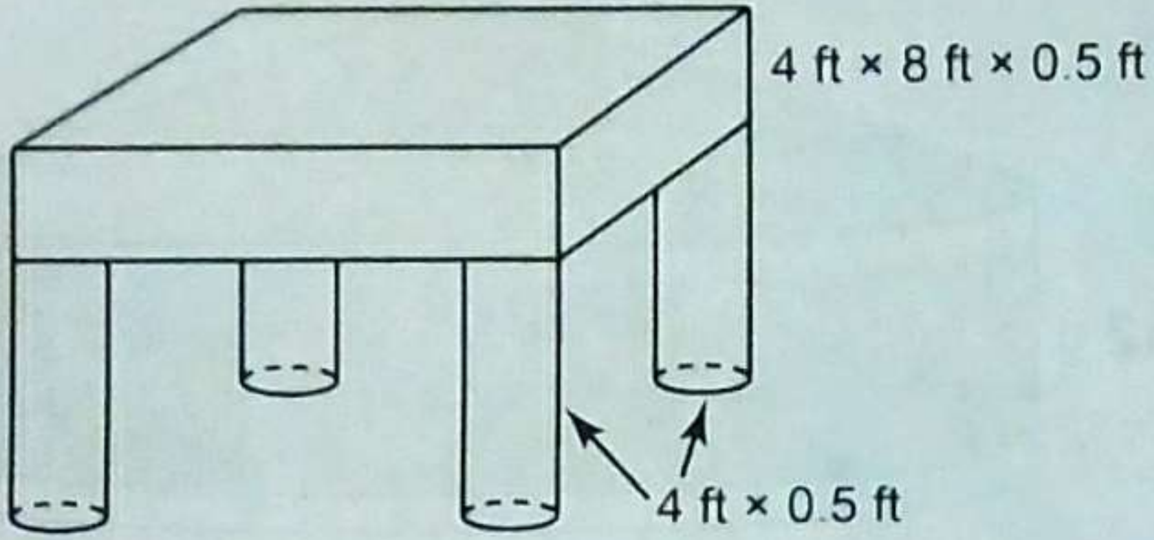
4. A theater company built the stage shown below. What is the volume of the figure?



A. 1,632 cm^3
B. 2,316 cm^3
C. 5,312 cm^3
D. 5,472 cm^3

The front face of the stage can be broken into two rectangles, the base, which is 30 cm by 5 cm, and the step, which is 12 cm by 16 cm. The areas of the two sections are 150 cm^2 and 192 cm^2, respectively, for a combined area of 342 cm^2. The volume is that area multiplied by the depth of 16 cm, or 5,472 cm^3 (choice D).

The following is a sketch of a sturdy work table composed of a rectangular top, 4 ft wide × 8 ft long × 0.5 ft thick, and four cylindrical legs, each 4 ft long with a diameter of 0.5 ft.

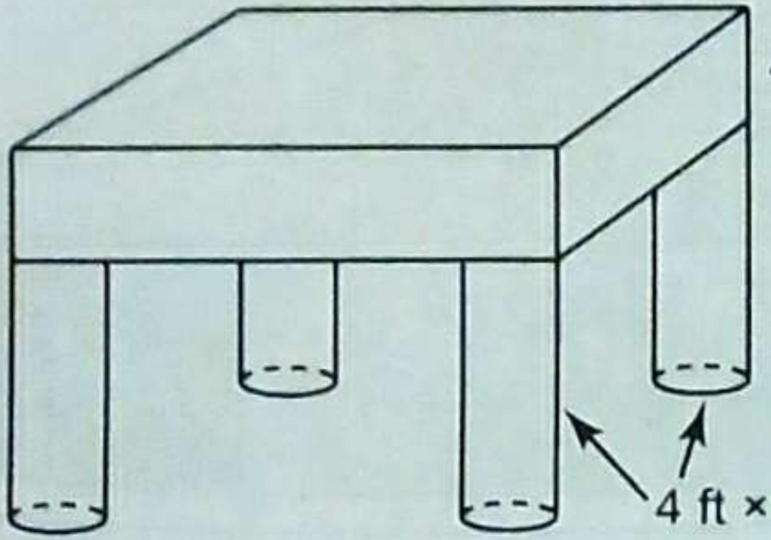


A. 16.0 cubic feet
B. 17.6 cubic feet
C. 19.1 cubic feet
D. 20.0 cubic feet

9. What is the total volume of the table, including the legs, to the nearest tenth of a cubic foot?

The volume of the table top is the product of its length, width, and thickness: (4.0)(8.0)(0.5) = 16 cubic feet. The volume of one leg is $\pi r^2 h$, where $r = 0.25$ feet and $h = 4$ feet: $V = (3.14)(.25)^2(4) = 0.785$ cubic feet. There are four legs so the total volume of the table and legs is (16) + 4(0.785) = 19.1 cubic feet (choice C).

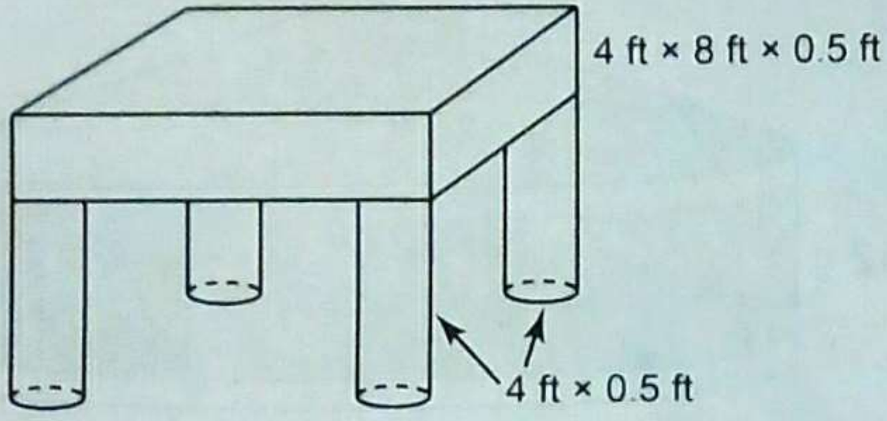
Volume of the legs only: 3.14 ft^3.



A. 377 pounds
B. 423 pounds
C. 800 pounds
D. 1,177 pounds

10. If the material used to make the table top weighs 50 lb per ft^3, and the material used to make the legs weighs 120 lb per ft^3, how much does the table weigh?

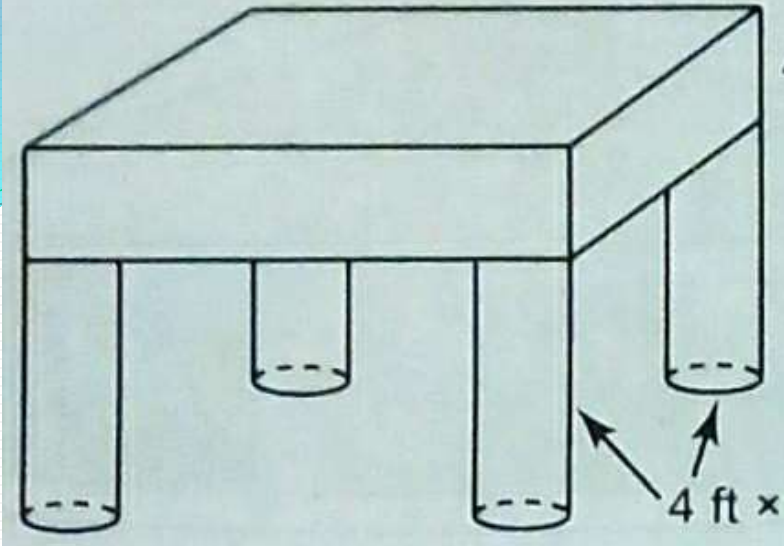
The weight of the table top is 50 pounds per cubic foot times the volume (16 cubic feet), or 800 pounds. The weight of the legs is 120 pounds per cubic foot times the total volume of the legs (3.14 cubic feet), or 376.8 pounds, which rounds to 377 pounds. The total weight is the sum of the two, or 1,177 pounds (choice D).



A. 101 square feet
B. 76 square feet
C. 63 square feet
D. 51 square feet

11. What is the combined surface area of the table top and legs, expressed to the nearest square foot?

The area of the top surface of the table is 4 × 8 = 32 square feet. The area of the front edge is 4 × 0.5 = 2 square feet, and the area of the side edge is 8 × 0.5 = 4 square feet. Combining those gives 38 square feet. For each of those surfaces, there are opposing surfaces not visible in the figure, so the total surface area of the table top is twice that, or 76 square feet. The surface area of each table leg is $2\pi rh$, where $r = 0.25$ and $h = 4$. Substitute so that 2(3.14)(.25)(4) = a surface area of 6.28 square feet per leg, or 25.12 square feet, which rounds to 25 square feet. Add the surface area of the table (76 square feet) to the surface area of the legs (25 square feet) to get a total surface area of 101 square feet.



A. 50 square feet
B. 57 square feet
C. 69 square feet
D. 101 square feet

12. A varnish is applied to the top of the table, the four sides of the table, and the lateral surfaces of the legs. To the nearest square foot, what is the total area of the table that is varnished?

The total area that is varnished is the top of the table, measuring 4 × 8 = 32 square feet; two sides of the table, each measuring 8 × 0.5 = 4 square feet for a total of 8 square feet; two sides of the table, each measuring 4 × 0.5 = 2 square feet for a total of 4 square feet; and 4 lateral surfaces, each measuring 2 × 3.14 × .25 × 4 = 6.28 square feet for a total of 25.12 square feet. Add to find the total area of the table that is varnished: 32 + 8 + 4 + 25.12 = 69.12 square feet, which rounds downward to 69 square feet.