Concluding Math

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Slope in Solving Geometric Problems (SiSGP)

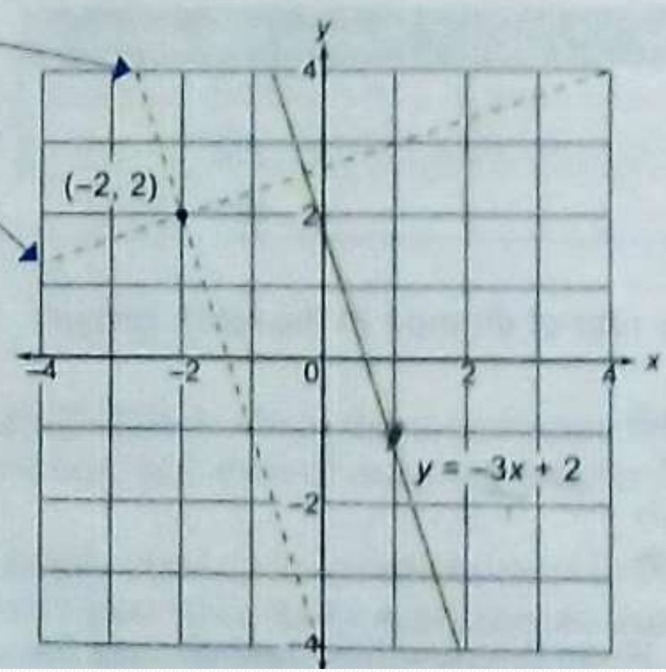
If two lines have the same slope, they are **parallel** to one another. If two lines have slopes that are negative inverses of each other—for example, one being −3 and the other $\frac{1}{3}$—then the lines are **perpendicular** to each other.

Some geometric figures, such as squares and rectangles, are made up of line segments that are parallel or perpendicular to each other. Understanding slope can help you to analyze such figures.

a The equations of the dashed lines can be deduced using the rules regarding slopes and the slope-point formula of a line: $(y - y_1) = m(x - x_1)$, where m is the slope and the point (x_1, y_1) is a point on the line.

b A good check to determine whether the slopes of perpendicular lines have been correctly determined: the product of the two slopes must equal −1. In this case, −3 multiplied by $\frac{1}{3}$ equals −1, which confirms that the lines are perpendicular.

The following graph shows a solid line with equation $y = -3x + 2$ and a point (−2, 2). The dashed lines pass through the point and are either parallel or perpendicular to the given line.



1. What is the equation for the line parallel to the given line? $y = -3x + 2$

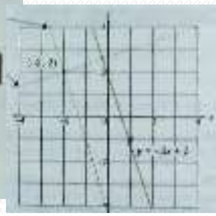
A. $(y - 2) = -3(x + 2)$

B. $(y - 2) = -\frac{1}{3}(x + 2)$

C. $(y + 2) = -3(x - 2)$

D. $(y + 2) = -\frac{1}{3}(x - 2)$

The equation for the given line is in slope-intercept form, with the slope specified as −3. Lines parallel to the given line also will have a slope of −3. Using the slope-point formula, with the slope of −3 and the point (−2, 2), $(y - 2)$, the equation for the parallel line is $y = -3(x + 2)$ (choice A). Choices B and D have the inverse of the correct slope, and choices C and D have the x- and y-values of the specified point switched.



2. What is the equation for the line perpendicular to the given line?

A. $(y - 2) = \frac{1}{3}(x + 2)$

B. $(y - 2) = 3(x + 2)$

C. $(y + 2) = \frac{1}{3}(x + 2)$

D. $(y + 2) = -3(x + 2)$

Again, the slope of the given line is −3. Lines perpendicular to the given line will have slopes that are the negative inverse of −3, or $+\frac{1}{3}$. Again, using the slope-point form of the equation with the specified point (−2, 2), one gets $(y - 2) = \frac{1}{3}(x + 2)$ (choice A). Choices B and D have the inverse of the correct slope, and choice C has the incorrect *y*-value.

A linear equation is represented by $y = \frac{1}{2}x + 3$.

The graph of which equation would be parallel to that of the equation above?

A. $y = \frac{1}{3}x + 2$

B. $y = \frac{1}{2}x - 3$

C. $y = -2x + 3$

D. $y = x + 3$

The given equation is already in slope-intercept form, $y = mx + b$, and so the slope of the given line is $m = \frac{1}{2}$. Any line parallel to the given line will have that same slope. Choice B is the only equation that meets that condition. Choice A swaps the 2 and the 3 in the given equation. Choice C represents a line perpendicular to the given line. Choice D has the same *y*-intercept, but twice the slope.

Graphing Quadratic Equations (GQE)

Quadratic equations are equations set in the form of $ax^2 + bx + c = 0$, where a is not zero. The word *quadratic* comes from the root *quad*, meaning "square." Therefore, a quadratic equation includes a squared variable, x^2. The solution to a quadratic equation may be found using the quadratic formula of $x = \frac{-b}{2a}$.

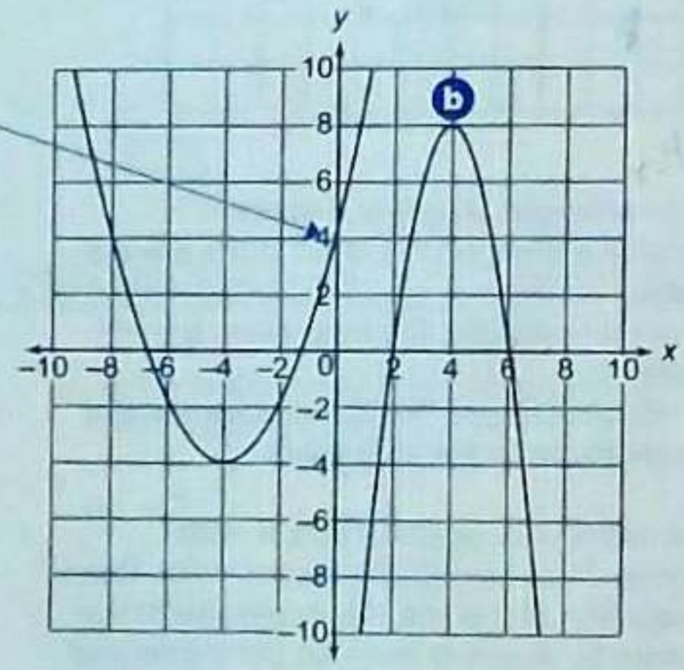
Characteristics of quadratic equations include zero, one or two points where the plot of such an equation crosses the x-axis, one point where it crosses the y-axis, either a maximum (when $a < 0$) or a minimum (when $a > 0$), and symmetry with respect to that maximum or minimum.

Coefficients of an equation—a, b, and c—can quantify these characteristics. For example, larger values of a will contract a curve, while smaller values of a will expand it. Negative values of a will turn it upside down.

a Symmetry can be used to identify additional points on the curve. For example, the left-hand curve goes through the point (0, 4). This is four units to the right of the minimum, which has an x-value of −4. Through symmetry, you can determine a point on the curve, with $y = 4$, four units to the left of the minimum, at $x = -4 - 4 = -8$, giving the point (−8, 4).

b The larger the magnitude of a, the steeper the curve will be. The right-hand curve has a of magnitude 2, while the left-hand curve has a of magnitude $\frac{1}{2}$. The right-hand curve shows a more rapid change in y as one moves away from the maximum than is the case for changes in y as one moves away from the minimum of the left-hand curve.

Two quadratic equations are plotted on the graph below. The one to the left is $y = \frac{1}{2}x^2 + 4x + 4$, while the one to the right is $y = -2x^2 + 16x - 24$.

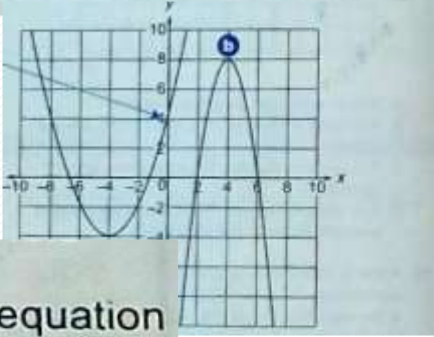


1. At what y-value does the right-hand curve cross the y-axis?

A. −16
B. −24
C. −32
D. −40

The curve crosses the y-axis at $y = c$, where c is the constant term in the equation. In the case of the right-hand curve, that is $y = -24$ (choice B).

left is $y = \frac{1}{2}x^2 + 4x + 4$, while the one to the right is $y = -2x^2 + 16x - 24$.



2. At what x-values does the left-hand curve cross the x-axis, expressed to the nearest tenth?

A. $x = -6.8$, $x = -1.2$
B. $x = -6.7$, $x = -1.2$
C. $x = -6.8$, $x = -1.1$
D. $x = -6.7$, $x = -1.1$

Setting the left-hand expression equal to zero and multiplying through by 2 gives: $x^2 + 8x + 8 = 0$. The equation does not factor, so one must use the quadratic formula $\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$. The resulting x-values are:

$$x = \frac{-8 \pm \sqrt{64 - 32}}{2} = \frac{-8 \pm \sqrt{32}}{2} = \frac{-8 \pm 5.656}{2} = \frac{-4 \pm 2.828}{1},$$

which leads to rounded values of −6.8 and −1.2 (choice A). Remaining answer choices are uniformly distributed, nearby points.

3. Which value of y corresponds to the point where the curve defined by $y = 2x^2 - 5x + 3$ crosses the y-axis?

A. $y = -5$
B. $y = -3$
C. $y = 3$
D. $y = 5$

The curve crosses the y-axis when $x = 0$. For this equation, substituting $x = 0$ into the equation of $y = 2x^2 - 5x + 3$ gives $y = +3$ (choice C).

Evaluation of Functions (EoF)

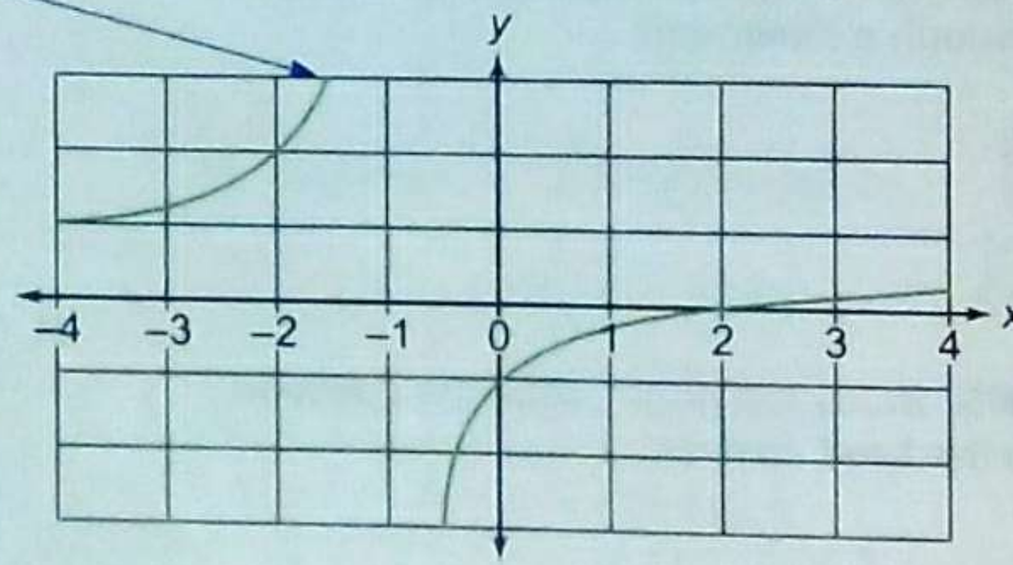
A **function** includes three parts: the input, the relationship, and the output. For example, an input of 8 and a relationship of x 7 produces an output of 56 ($8 \times 7 = 56$). In the function $f(x) = x^2$, f is the function, x is the input, and x^2 is the output. The function $f(x) = x^2$ shows that the function f takes the x and squares it. So an input of 8 would result in an output of 64: $f(8) = 8^2$.

Functions generally have one output (y-value) for each input (x-value). Functions and their properties or traits may be displayed in graphs, tables, and algebraic expressions. These include traits of quadratic functions: intercepts, maxima and minima, and symmetries. Calculation of isolated points located near intercepts and points where a function is undefined provide additional information about where a function is positive, negative, increasing, or decreasing, and estimates of where the function has relative maxima and minima.

a Points where functions are undefined—such as x-values at which a denominator in a function becomes zero—are reflected in rapid changes in the function to y-values on either side of the graph.

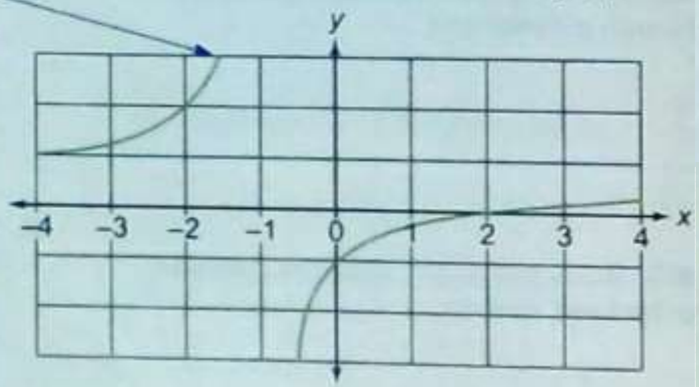
b Rational functions, which are ratios of polynomials, can feature both x-intercepts and points at which a function is undefined. These points are apparent in graphical representations of the functions, such as those in the graph to the right, and often can be determined from equations.

b The following graph plots the function $y = \dfrac{x-2}{x+1}$:



b

The following graph plots the function $y = \frac{x-2}{x+1}$.



1. At what x-value does the above function intercept the x-axis?

A. $x = -2$
B. $x = -1$
C. $x = 0$
D. $x = 2$

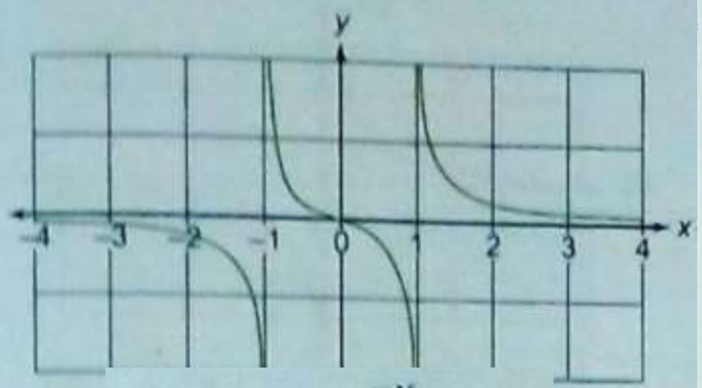
The value at which the curve crosses the x-axis can be determined from the graph or by noting that the numerator in $y = \frac{x-2}{x+1}$, $x - 2$, goes to 0 when $x = 2$ (choice D). Choice A is the opposite of the correct answer, and also the y-value of the y intercept. Choice B is the x-value at which the function is undefined. Choice C is the next integer in the sequence following choices A and B.

2. For what x-value is the above function undefined?

A. $x = -2$
B. $x = -1$
C. $x = 0$
D. $x = 2$

Inspection of the function shows that the denominator in $y = \frac{x-2}{x+1}$, $x + 1$, goes to 0 when $x = -1$ (choice B). This is consistent with the graphical representation of the function, where the curves show radical behavior as they approach $x = -1$, both from below and above. The remaining choices parallel those of question 1.

3. Which equation corresponds to the graph below?



A. $y = \dfrac{-x}{(x+2)(x-2)}$

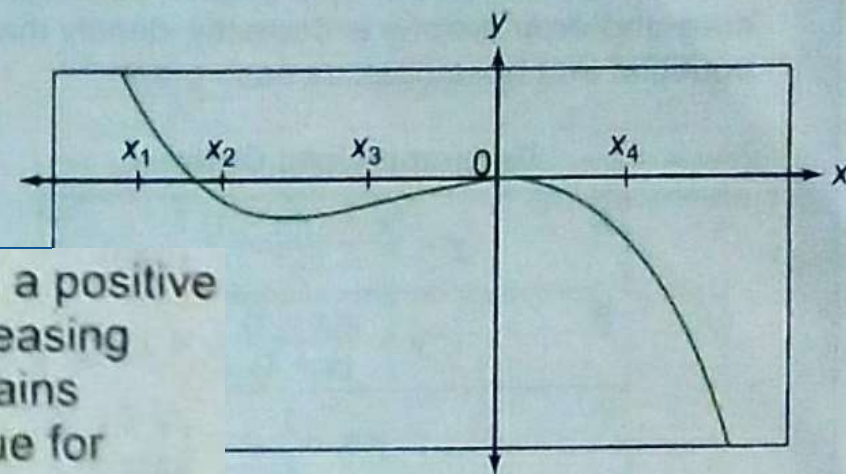
B. $y = \dfrac{-x}{(x+1)(x-1)}$

C. $y = \dfrac{x}{(x+2)(x-2)}$

D. $y = \dfrac{x}{(x+1)(x-1)}$

The graph shows that, for large values of x (such as 3, 4, and so on), the function is positive. That eliminates choices A and B since, for large values of x, the numerators of those choices are both negative, and the denominators of both are positive. The graph also shows that the function is undefined (denominator goes to zero) at $x = 1$ and $x = -1$, indicating that the denominators go to 0 for those values of x. That eliminates choice C, leaving choice D as the only possible option.

The following graph is a plot of the function $y = -x^3 - 2x^2$. Four x positions are labeled.



5. At which of the specified x-values is y positive?

A. x_1
B. x_2
C. x_3
D. x_4

By inspection of the graph, the only x-value with a positive corresponding y-value is x_1. The function is increasing slightly at x_2 and at x_3 (choices B and C), but retains negative values in both cases. The function value for choice D is negative.

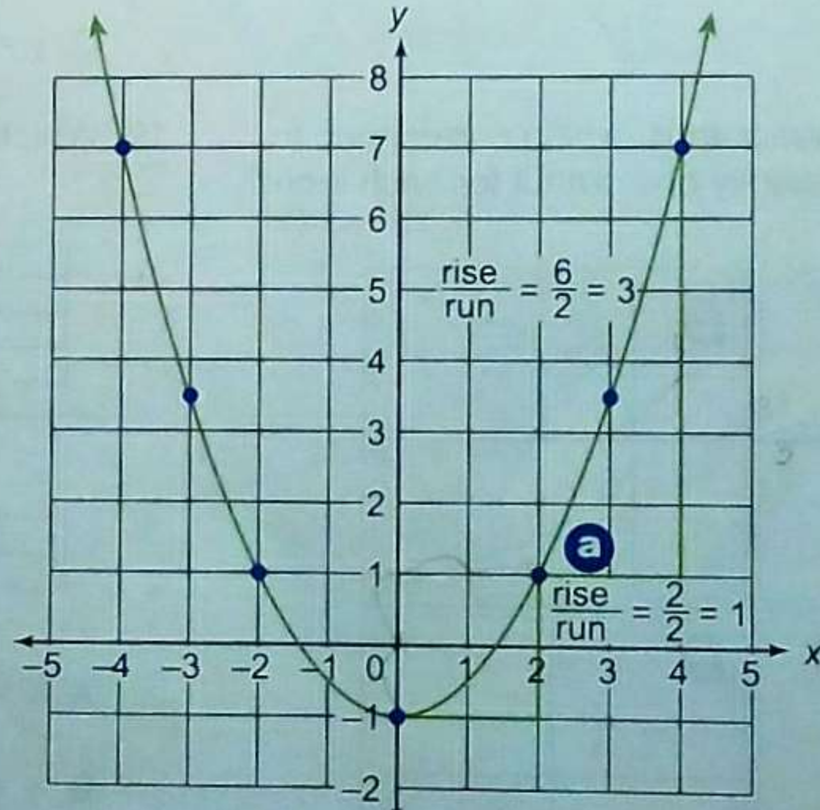
Comparison of Functions (CoF)

Functions can be represented by sets of ordered pairs, in tables, in graphs, algebraically, or by verbal descriptions. Two or more functions can be compared based on their slopes or rates of change, intercepts, the locations and values of minimums and maximums, and other features. You can compare two linear functions, two quadratic functions, or a linear function and a quadratic function.

Linear functions are represented by graphs that are straight lines. The slope of the line is the rate of change of the function. The rate of change is constant, meaning that for any two points on the line, the slope is the same. **Quadratic functions** are represented by graphs that are parabolas. Quadratic functions do not have a constant rate of change. You can find the **average rate of change** of a function over a particular interval by finding the ratio of the vertical change to the horizontal change between two points.

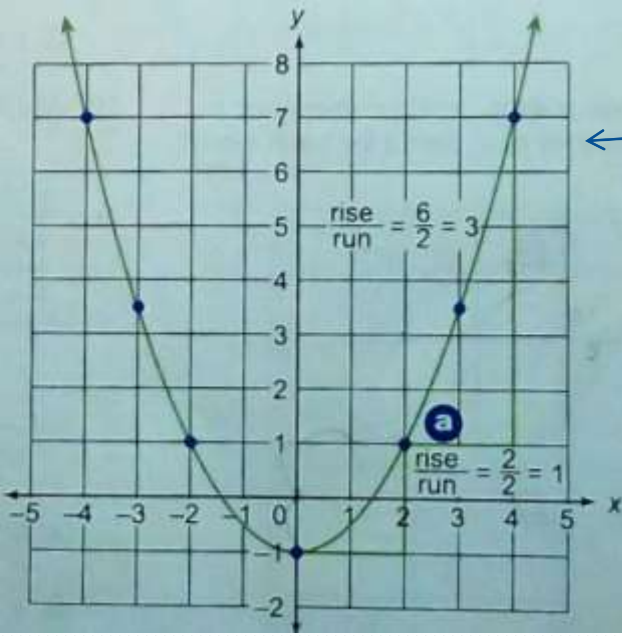
a The function represented in the graph is a quadratic function. The average rate of change from $x = 0$ to $x = 2$ $\frac{2}{2} = 1$. The average rate of change from $x = 2$ to $x = 4$ is $\frac{7-1}{2} = 3$.

b The function represented in the table has a constant rate of change of 2. So, the function represented in the table has a greater rate of change than the average rate of change for the function represented in the graph for $x = 0$ to $x = 2$ and a lesser rate of change for $x = 2$ to $x = 4$.



x	y
−4	−4
−3	−2
−2	0
−1	2
0	4
1	6
2	8
3	10
4	12

b



1. Which function below has a lesser rate of change than the average rate of change of the quadratic function over the interval $x = 0$ to $x = 3$?

A. $f(x) = 2x - 1$
B. $f(x) = 0.5x + 3$
C. $f(x) = 7x + 2$
D. $f(x) = 1.5x - 4$

At $x = 0$, the quadratic function has a value of -1. At $x = 3$, the quadratic function has a value of 3.5. So, the average rate of change of the function is $\frac{3.5}{3} \approx 1.17$. The rate of change of a linear function represented algebraically, in function notation, is given by the x-coefficient. So, compare the x-coefficient of the function in each answer choice to 1.17. Only answer choice B has an x-coefficient (0.5) that is less than 1.17.

2. Over what interval is the rate of change of the quadratic function above the same as the rate of change of the function $f(x) = -2x - 3$?

A. $x = -4$ to $x = 0$
B. $x = -4$ to $x = -2$
C. $x = -3$ to $x = 2$
D. $x = 0$ to $x = 4$

The rate of change of a linear function represented in function notation is the x-coefficient. So, the rate of change of $f(x) = -2x - 3$ is -2. Find the average rate of change of the quadratic equation over each interval and compare to -2. In answer choice A, $f(-4) = 7$ and $f(0) = -1$. So, the average rate of change is $\frac{7 - (-1)}{-4 - 0} = \frac{8}{-4} = -2$. So, the average rate of change (-2) of the quadratic function over the interval $x = -4$ to $x = 0$ is the same as the rate of change of the function $f(x) = -2x - 3$. In answer choice B, $f(-4) = 7$ and $f(-2) = 1$, so the average rate of change is $\frac{7 - 1}{-4 - -2} = \frac{6}{-2} = -3$. In answer choice C, $f(-3) = 3.5$ and $f(2) = 1$, so the average rate of change is $\frac{3.5 - 1}{-3 - 2} = \frac{2.5}{-5} = -0.5$. In answer choice D, $f(0) = -1$ and $f(4) = 7$, so the average rate of change is $\frac{7 - (-1)}{4 - 0} = \frac{8}{4} = 2$

17. A linear function has a rate of change of −0.5 and a *y*-intercept of 5. Which statement is true?

x	*y*
-4	6
-2	5
0	4
2	3
4	2

A. The function has the same rate of change and *y*-intercept as the function represented in the table.

B. The function has a greater rate of change and *y*-intercept as the function represented in the table.

C. The function has a greater rate of change and the same *y*-intercept as the function represented in the table.

D. The function has the same rate of change and a greater *y*-intercept than the function represented in the table.

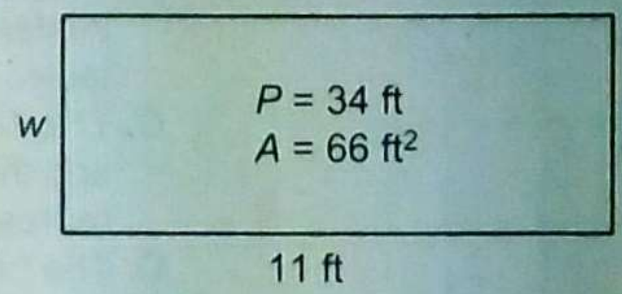
The rate of change of a linear function is the ratio of the vertical change to the horizontal change. The function represented in the table has a rate of change of $\frac{5-6}{-2-(-4)} = \frac{-1}{2} = -0.5$. Therefore, the given function has the same rate of change as the function represented in the table. The *y*-intercept of a function is the *y*-value when the *x*-value is 0, so the *y*-intercept of the function represented in the table is 4. Therefore, the given function has a greater *y*-intercept (5) than the function represented in the table.

Triangles & Quadrilaterals (T&Q)

A **triangle** is a closed three-sided figure with three angles or corners. The **area** of a triangle is $\frac{1}{2}bh$, where b is the base and h is the height. The **perimeter** of a triangle is the sum of its side lengths.

A **quadrilateral** is a closed four-sided figure with four angles. The sides of a quadrilateral may or may not be congruent or parallel. The perimeter of a quadrilateral is the sum of its side lengths. If the quadrilateral has two or more congruent sides, a formula can be used to find its perimeter. Use a formula to find the area of a rectangle ($A = lw$), a square ($A = s^2$), or a parallelogram ($A = bh$).

a To find the missing measure, use inverse operations to isolate the variable. Then perform the same operations on each side of the equals sign to keep the equation balanced.



Rectangle

Perimeter = $2l + 2w$

$34 = 2(11) + 2w$

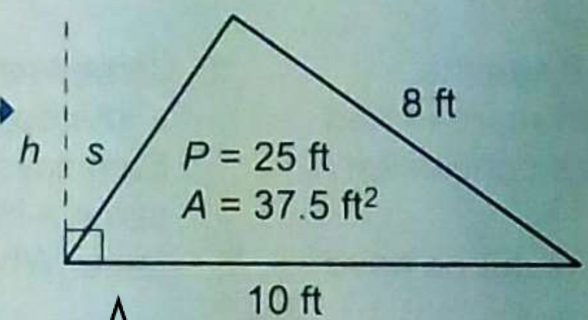
$34 = 22 + 2w$

a $12 = 2w \rightarrow w = 6$ ft

Area = lw

$66 = 11w \rightarrow w = 6$

b The height of an acute or isosceles triangle, or of a parallelogram, may be shown as a line perpendicular to the base. The line can be shown inside or outside of the figure.



Triangle

Perimeter = side + side + side

$25 = 10 + 8 + s$

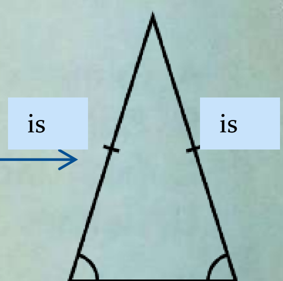
$25 = 18 + s \rightarrow s = 7$

Area = $\frac{1}{2}bh$

$37.5 = \frac{1}{2}(10)(h)$

$37.5 = 5h \rightarrow h = 7.5$

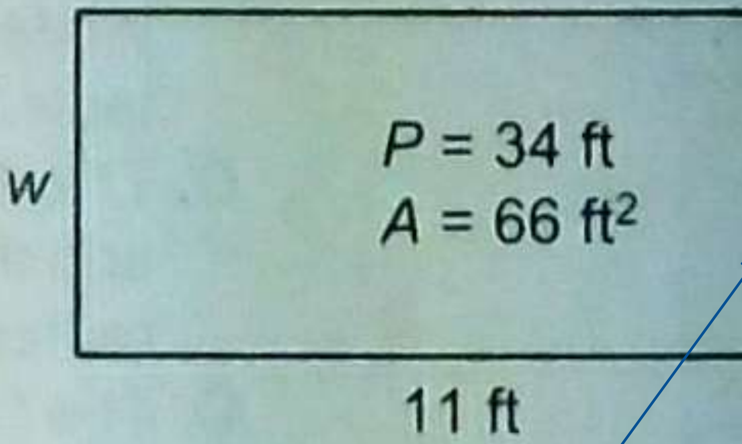
c An isosceles triangle has at least 2 congruent sides. Since the perimeter of a triangle is the sum of its side lengths, you can find the length of one congruent side by subtracting the length of the base from the perimeter and dividing the difference by 2.



P = 20

b = 4

is = (20-4)/2 = 8



A. 8.5 ft
B. 16.5 ft
C. 17 ft
D. 34 ft

Rectangle

Perimeter = $2l + 2w$

$34 = 2(11) + 2w$

$34 = 22 + 2w$

a $12 = 2w \rightarrow w = 6$ ft

Area = lw

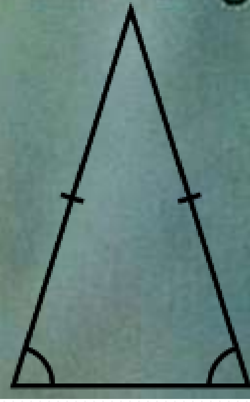
$66 = 11w \rightarrow w = 6$

What is the side length of a square with the same perimeter as the rectangle above?

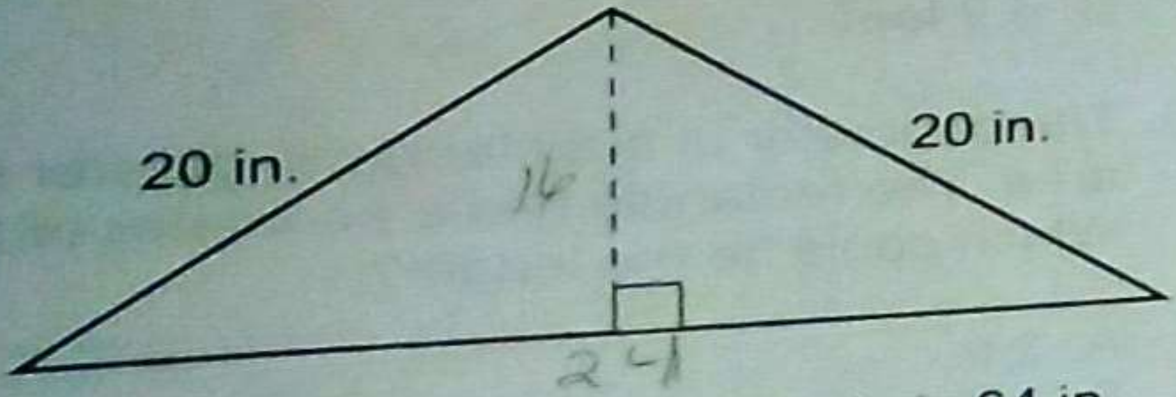
The perimeter of a square is given by the formula $P = 4s$. The perimeter of the square is 34 feet, so $34 = 4s$. Divide each side by 4: $s = 8.5$. Answer choice B is the result of dividing the area of the rectangle by 4. Answer choice C is the result of dividing the perimeter of the rectangle by 2. Answer choice D is the perimeter of the square.

An isosceles triangle with a base of 8 cm has a perimeter of 28 cm. Which could be the length of each of the other two sides?

A. 6 cm
B. 10 cm
C. 18 cm
D. 20 cm



The perimeter of a triangle is the sum of its side lengths. Since an isosceles triangle has two congruent sides, $P = b + 2s$. Substitute 28 for P and 8 for b: $28 = 8 + 2s$. Subtract 8 from each side: $20 = 2s$. Divide each side by 2: $s = 10$. Answer choice A is the result of dividing 28 by 2 and then subtracting 8. Answer choice C is the result of adding 28 + 8 and dividing the sum by 2. Answer choice D is the sum of the lengths of the two sides.



The perimeter of the triangle is 64 in.
The area of the triangle is 192 in.^{2}

What is the base of the triangle?

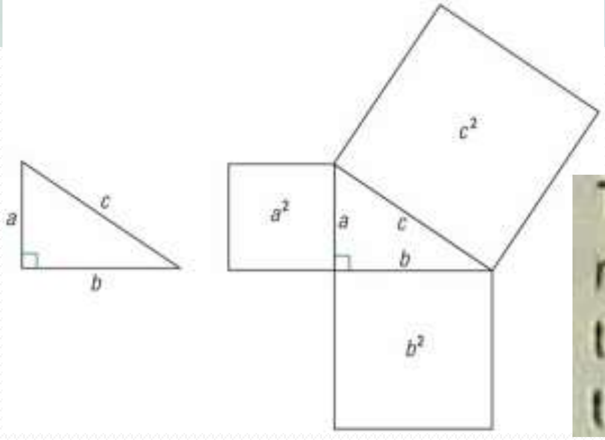
What is the height of the triangle? in.

The perimeter of a triangle is the sum of the lengths of its sides. Since the perimeter is 64 inches, $20 + 20 + b = 64$. Combine like terms and subtract 40 from each side: $b = 24$.

The area of a triangle is $\frac{1}{2}bh$. The base of the triangle is $64 - 20 - 20 = 24$ inches, so $192 = \frac{1}{2}(24)h$. Multiply: $192 = 12h$. Divide each side by 12: $h = 16$.

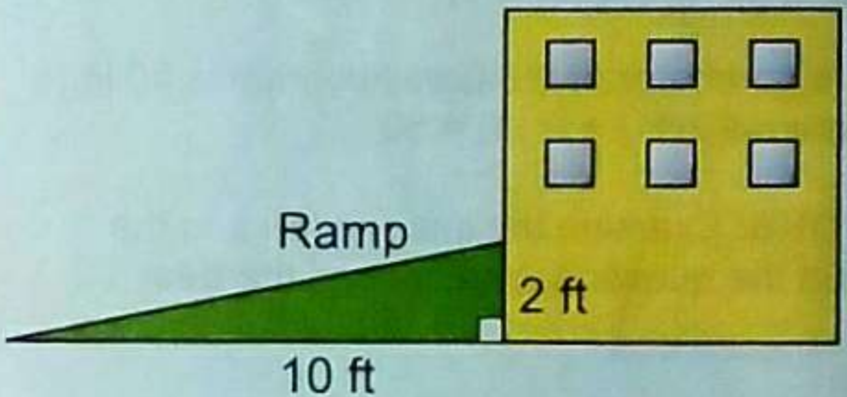
Pythagorean Triangle (PT)

As you know, a **right triangle** has a right angle. The legs (shorter sides) and **hypotenuse** (longer side) of a right triangle have a special relationship that can be described by the **Pythagorean Theorem**. It states that, in any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. It is expressed in equation form as $a^2 + b^2 = c^2$. You can use this theorem to find a missing length of a right triangle.



The measurements of the legs of the right triangle are given. Solve for the hypotenuse to find the length of the ramp.

A ramp was built to add wheelchair access to a public building. The ramp rises 2 feet, as shown in the diagram below.



ⓐ 1. If the lower edge of the ramp is 10 feet from the base of the building along level ground, what is the approximate length, in feet, of the ramp?

(or 12 ft for the second question)

A. 9.2
B. 9.6
C. 9.8
D. 10.2

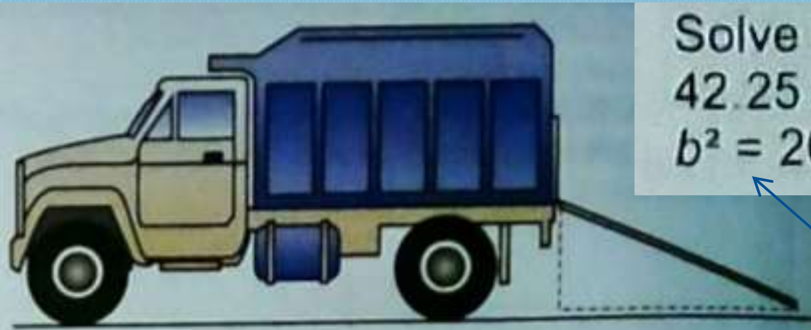
Solve $10^2 + 2^2 = c^2$ to find that $c^2 = 104$ and $c \approx 10.2$ ft.

ⓑ 2. The owners of the building are remodeling the front entrance. They would like to modify the ramp so that it begins 12 feet from the building. What will be the length of this new ramp?

A. 11.8 ft
B. 12.2 ft
C. 12.4 ft
D. 12.5 ft

To solve question 2, substitute 12 for 10 and solve for the hypotenuse. Remember that the hypotenuse is always the longest side of a right triangle.

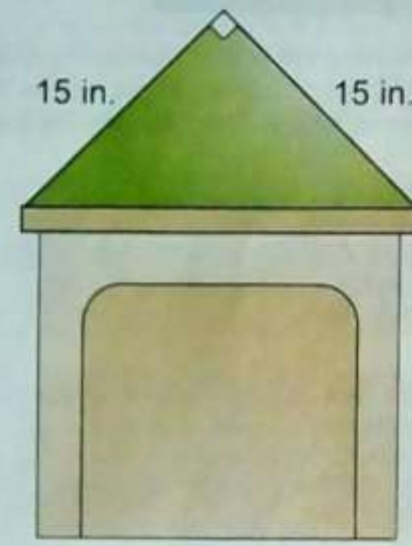
Solve $12^2 + 2^2 = c^2$ to find that $c^2 = 148$ and $c \approx 12.17$ ft, which rounds to 12.2 ft.



13. A 7.9 foot ramp runs from the back of a truck to the ground. If the ramp meets the ground 6.5 feet away from the truck, about how many feet off the ground is the ramp?

A. 2.0
B. 4.5
C. 7.1
D. 10.2

Solve $6.5^2 + b^2 = 7.9^2$ to find the other leg so that $42.25 + b^2 = 62.41$. Subtract 42.25 from 62.41 so that $b^2 = 20.16$ and $b \approx 4.49$, which rounds to 4.5.

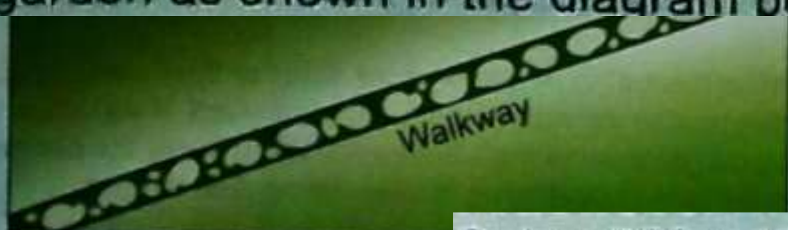


22. What is the approximate width of the dollhouse in inches?

A. 17.5
B. 18.6
C. 19.2
D. 21.2

Solve $15^2 + 15^2 = c^2$ to find the width of the dollhouse. Add $225 + 225 = c^2$ so that $c^2 = 450$. Therefore, $c \approx 21.2$.

Henry is building a walkway through a rectangular garden as shown in the diagram below.



17. If the length of the garden is 30 yards and the width of the garden is 17 yards, what is the approximate length of the walkway in yards?

A. 34.5
B. 24.7
C. 21.4
D. 13.3

Solve $30^2 + 17^2 = c^2$ to find the walkway in the garden. Add $900 + 289 = c^2$ so that $c^2 = 1{,}189$. Therefore, $c \approx 34.48$, which rounds to 34.5.

Polygons (P)

A **polygon** is any closed figure with three or more sides. A polygon is named according to its number of sides. For example, a pentagon has five sides, a hexagon has six sides, and an octagon has eight sides. The perimeter of a **regular polygon** is the product of its side length and number of sides. The perimeter of an **irregular polygon** is the sum of its side lengths.

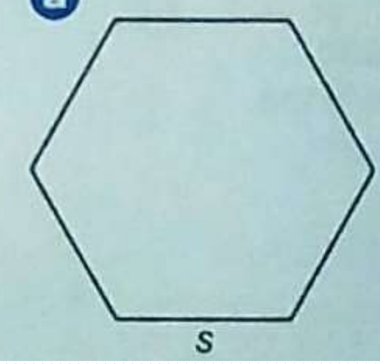
If you know the perimeter, you may be able to determine the length of one or more sides of the figure. If you know the perimeter and side length of a regular polygon, you can determine the number of sides. If you know the perimeter and number of sides of a regular polygon, you can determine the side length. If you know the perimeter and some side lengths of an irregular polygon, you may be able to determine the remaining side lengths.

a The perimeter of a regular polygon is ns, where n is the number of sides and s is the side length. If you know the perimeter, work backward to find the side length: $s = p \div n$.

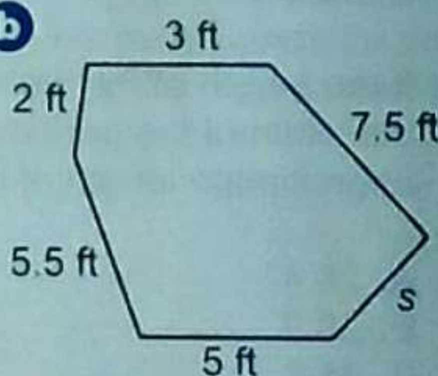
b The perimeter of an irregular polygon is the sum of its side lengths. Subtract the known side lengths from the perimeter to find the remaining side length.

Each figure has a perimeter of 27 feet.

a



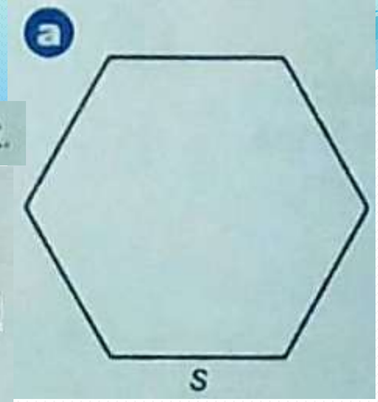
b



What is the length of each side of the regular hexagon?

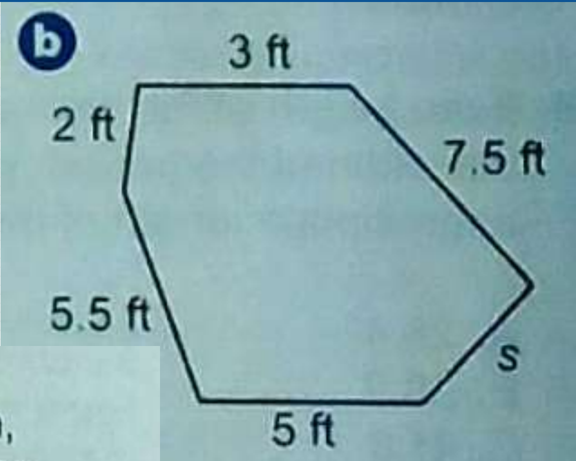
A. 3.375 feet
B. 4.5 feet
C. 5.4 feet
D. 21 feet

Each figure has a perimeter of 27 feet.



The perimeter of a regular polygon is the product of its side length and number of sides. The figure has 6 sides, so $s = 27 \div 6 = 4.5$ feet. Answer choice A is the result of finding the side length of a figure with 8 sides. Answer choice C is the result of finding the side length of a figure with 5 sides. Answer choice D is the result of subtracting 6 from 27.

2. What is the difference, in inches, between the side length of the regular hexagon and the unknown side length of the irregular hexagon?

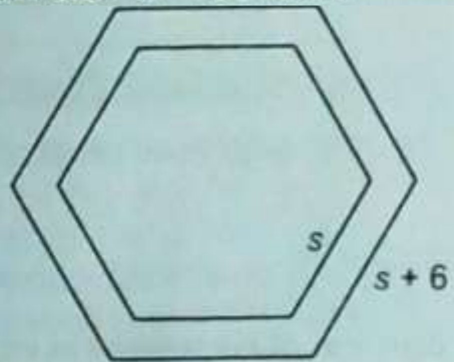


To answer question 2, multiply the number of feet by 12 to find the answer in inches.

The perimeter of an irregular polygon is the sum of its side lengths. To find an unknown side length, subtract the known side lengths from the perimeter: $s = 27 - (7.5 + 3 + 2 + 5.5 + 5) = 27 - 23 = 4$. The length of the side of the regular polygon is the perimeter divided by the number of sides: $27 \div 6 = 4.5$ feet. Subtract: $4.5 - 4 = 0.5$ feet. There are 12 inches in a foot, so 0.5 feet = (0.5)(12) = 6 inches. Answer choice A is the result of converting 0.5 feet to 5 inches. Answer choice C is the result of incorrectly calculating the length of the unknown side in either figure. Answer choice D is the length of the unknown side in the irregular figure.

A. 5 in.
B. 6 in.
C. 12 in.
D. 48 in.

. Daniel installed a swimming pool in the shape of a regular hexagon. He surrounded the swimming pool with a deck and built a hexagonal fence around the deck. The outside edge of the deck is 6 feet wider than the side of the pool. The fence has a perimeter of 108 feet.



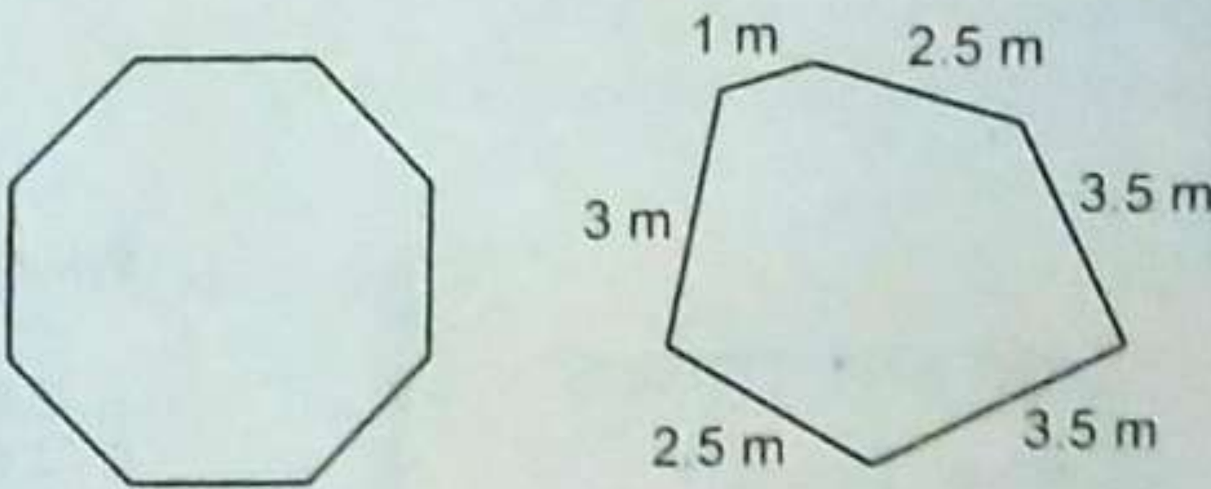
How long is each side of the swimming pool?

A. 12 ft
B. 18 ft
C. 24 ft
D. 17 ft

The perimeter of a regular polygon is the product of its side length and number of sides. The fence has a perimeter of 108 feet and it has 6 sides, so the length of each side is 108 ÷ 6 = 18 feet. The length of the side of the fence is 6 feet greater than the length of the side of the pool, so the side of the pool is 18 − 6 = 12 feet. Answer choice B is the side length of the fence. Answer choice C is the result of adding 6 to the side length of the fence. Answer choice D is the result of subtracting 6 from 108 and dividing the difference by 6.

each side of

22. The figure on the left is a regular polygon.



If the perimeter of the regular polygon is 1.5 times the perimeter of the irregular polygon, what is the side length of the regular polygon?

A. 2 m
B. 3 m
C. 4 m
D. 5 m

The perimeter of an irregular polygon is the sum of its side lengths. Add: 3 + 1 + 2.5 + 3.5 + 3.5 + 2.5 = 16 m. Multiply the perimeter of the irregular polygon by 1.5 to find the perimeter of the regular polygon: (1.5)(16) = 24 m. Since the regular polygon has 8 sides, divide the perimeter by 8 to find the side length: 24 ÷ 8 = 3 m.

Circles (C)

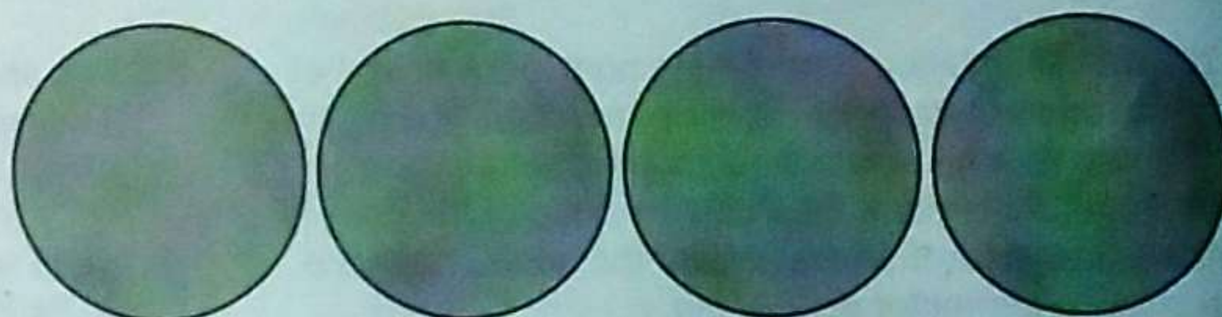
A **circle** is a closed figure with no sides or corners. All points on a circle are equidistant from the center. The distance from the center of a circle to any point on the circle is called the **radius**. The **diameter** is the distance across a circle through its center. The diameter is always twice the radius. The distance around a circle is known as its **circumference**.

To find a circle's circumference, use the formula $C = \pi d$. To find a circle's area, use the formula $A = \pi r^2$. You may find a circle's circumference or area if you know either its radius or its diameter. If you know the radius of a circle, you may double it to find the diameter. If you know the diameter of a circle, you may divide it by 2 to find the radius.

a For both questions, you know the circumference and want to find the diameter. Use the formula for circumference and then work backward.

b Note that the information provided is in *inches*. For question 2, multiply the number of stones by the diameter of a stone. Then divide by 12 (inches in a foot) to find the number of *feet*.

Elizabeth created a path through her garden with identical round paving stones like those shown below. The circumference of each stone is 25.9 inches.



What is the approximate diameter of each stone in inches?

Elizabeth created a path through her garden with identical round paving stones like those shown below. The circumference of each stone is 25.9 inches.

A. 2.87
B. 4.13
C. 6.48
D. 8.25

The circumference of a circle is the product of its diameter and π. Substitute 25.9 for C in the formula $C = \pi d$ and solve for d: $25.9 = 3.14d$. Divide each side by 3.14: $d \approx 8.25$ inches.

If Elizabeth uses 35 stones, about how many feet long will her garden path be?

A. 24
B. 76
C. 92
D. 289

If Elizabeth uses 35 stones, the total length of the stones will be $35 \times 8.25 = 289$ inches. There are 12 inches in 1 foot, so divide the number of inches by 12: $289 \div 12 = 24.1$ ft. So, the garden path will be about 24 feet.